

MAT - 132, Midterm Summer - 2005  
6:00 pm to 7:30 pm

July 28, 2005

**Answer Question no 1 and any two from the rest. All the questions carry equal credit. Indicate clearly which questions you want to be graded. This is a closed book closed notes exam. No calculator or communication devices allowed.**

1. Answer if the following statements are true or false with justification. If the statement is true supply a sketch of the proof. If the statement is false, provide with a counter example.

a.  $\int_{-\infty}^{\infty} \frac{x}{1+x^2} dx = 0$

b. If  $f : [3, 5] \rightarrow \mathbf{R}$  and  $\int_3^5 f(x) dx = 46$ , then  $f$  is continuous on  $[3, 5]$ .

**c.**  $\int_0^3 \frac{dx}{x-1} = \ln 2.$

**d.**  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5} = 0.2.$

**2a.** Prove that,  $\int_0^\infty e^{-x^2} dx$  is convergent.

**2b.** First prove that,  $\int_0^a f(x)dx = \int_0^a f(a-x)dx$  for any  $f$ . Then use that to find  $\int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$ .

**3a.** Find the indefinite integral  $\int e^x \cos x dx$ .

**3b.** Prove that,  $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$ .

**4a.** Find the area enclosed between the following two curves:  $y = x^4 - x^2$  and  $y = 1 - x^2$ .

**4b.** Find the length of the following parametric curve:

$$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta + \theta\cos\theta), 0 \leq \theta \leq \pi.$$