

# Project 1

(due by 12/14/06 – 5:00pm)

*Discuss the problem below in a concise and precise essay, at most 5 typed pages long. Whenever you use a reference, quote it and do not copy. Use your own words.*

Consider the linear operator  $T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$  which has the matrix

$$\begin{bmatrix} -4 & 3 & 1 & -1 \\ -6 & 5 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

with respect to the standard basis.

**1.** Argue that there is a basis  $v_1, v_2, v_3, v_4$  of  $\mathbb{R}^4$  for which the matrix of  $T$  is upper-triangular by explicit construction, using the methods of Chapter 5. In particular, all eigenvalues are real. Give them and compute the matrix.

**2.** Find all invariant subspaces of  $T$ . Why is there no basis of eigenvectors, so  $T$  is not diagonalizable?

**3.** Finally, apply the Gram-Schmidt process to the above basis and construct an orthonormal basis  $e_1, e_2, e_3, e_4$  and the corresponding upper-triangular matrix for  $T$  according to Schur's Theorem (Corollary (6.27) in the text book.).