

## Math 310: Model Proof

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When you write a proof is it very important to explain clearly the logic of your argument; what are you assuming? what are you trying to prove? how do you justify each step?

Some of the proofs in the book are rather too short. Here is an example to explain what I mean. This is Prop 1.6 in the book. (The proof in the book is a bit longer than the first one below, but the explanation is still rather brief.)

**Proposition 1.6**  $(-1)v = -v$  for every  $v \in V$ .

*Proof.* For every  $v \in V$

$$v + (-1)v = 1v + (-1)v = (1 + (-1))v = 0v = 0.$$

Therefore  $(-1)v = -v$ . □

This proof is too short. For one, there is no explanation of the conclusion: “Therefore  $(-1)v = -v$ ”. This step is justified by Prop 1.3 that says that every element  $v$  has a unique additive inverse, i.e. if  $v + w = 0$  then  $w$  is the additive inverse  $-v$ . Secondly the steps in the first statement  $v + (-1)v = 1v + (-1)v = (1 + (-1))v = 0v = 0$  are not explained. These are justified by the familiar axioms of addition and multiplication together with Prop 1.4.

Later on in the semester it would be fine for you to use the axioms and results in Ch 1 without comment. In this first week it would be best to explain each step.

Here is a fuller version of the proof.

**Proposition 1.6**  $(-1)v = -v$  for every  $v \in V$ .

*Proof.* For every  $v \in V$

$$\begin{aligned} v + (-1)v &= 1v + (-1)v && \text{by the Mult. identity axiom} \\ &= (1 + (-1))v && \text{by the distributive properties} \\ &= 0v \\ &= 0 && \text{by Prop 1.4,} \end{aligned}$$

Therefore  $(-1)v = -v$  by Prop 1.3. □