Math 310: Review Exercises Sept 5, 2006

Ex 1. Review complex numbers.
(i) Calculate (with answers in the form $z = a + ib$)

$$(2+3i)(4-i), \quad (2-3i)^{-1}, \quad \frac{1-i}{2+3i}.$$

(ii) Recall that for the complex number $z = a + ib \in \mathbb{C}$, the modulus $|z|$ is defined to be $\sqrt{a^2 + b^2}$. Calculate $|2 + 3i|, |1 + i|$.
(iii) Check that $|(2+3i)(1+i)| = |2+3i||1+i|$., i.e. the modulus of a product is the product of the moduli.

Ex 2. Review matrices, especially matrix multiplication. Calculate
$A + B, A - C, AC, CB$ and BA (if they are defined) when

$$A := \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}, \quad B := \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & -2 \end{bmatrix}, \quad C := \begin{bmatrix} 0 & 1 & 5 \\ 2 & -1 & 1 \end{bmatrix}.$$

Some linear algebra in (real) 3-space $\mathbb{R}^3$: Definitions
A 3-**vector** $\mathbf{v}$ or (vector in $\mathbb{R}^3$) is an ordered triple of 3 real numbers, usually written horizontally. eg $\mathbf{v} = (1, 2, 3)$ but sometimes vertically: $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.
The **span** of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ consists of all linear combinations

$$\mathbf{v} := a_1\mathbf{v}_1 + \cdots + a_k\mathbf{v}_k, \quad a_i \in \mathbb{R}.$$

The vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are said to be **linearly independent** if and only if

$$a_1\mathbf{v}_1 + \cdots + a_k\mathbf{v}_k = 0 \text{ implies } a_i = 0 \ \ \forall i,$$

i.e. the only linear combination $\sum_i a_i\mathbf{v}_i$ that equals 0 is the trivial combination with all coefficients equal to 0. If $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are not linearly independent then they are said to be **linearly dependent**.
Note: The book gives very slightly different (but equivalent) definitions; we will discuss this in class.

Here are some questions that use these concepts. Try to make careful arguments, not just a list of scraps of calculations. (This is good practice for what is coming...)
Ex 3. (i) Let $\mathbf{v}_1 = (1, 2, 1), \mathbf{v}_2 = (1, 0, 1), \mathbf{v}_3 = (0, 1, -1)$. Do these vectors span $\mathbb{R}^3$?
(ii) Are these vectors linearly independent?

Ex 4. Same questions for the vectors $\mathbf{v}_1 = (1, -2, 1), \mathbf{v}_2 = (1, 0, -1), \mathbf{v}_3 = (0, 1, -1)$.

Ex 5. Show that the span of the vectors $\mathbf{v}_1 = (1, -2, -1), \mathbf{v}_2 = (1, 0, 1), \mathbf{v}_3 = (0, 1, 1)$ is the plane $x + y - z = 0$.