HW14

Sec 7.3

Prob 6

the characteristic eqn is $\lambda^2 - 7\lambda - 2 = 0$

roots are $\lambda_{1,2} = \dfrac{7 \mp \sqrt{57}}{2}$

eigenvector are obtained using,

$$E_{1,2} = \mathrm{Ker}\left[A - \lambda_{1,2} I\right]$$

$$\therefore E_{\lambda = \frac{7-\sqrt{57}}{2}} = \begin{bmatrix} -3 \\ \frac{\sqrt{57}-3}{2} \end{bmatrix}$$

similarly, find $E_{\lambda = \frac{7+\sqrt{57}}{2}}$

Prob 20

Upper Triangular matrix $\longrightarrow \lambda = 1, 1, 2$

for $\lambda = 2$, algebric multiplicity $= 1 \longrightarrow$ geometric multiplicity $= 1$

for $\lambda = 1$, " " $= 2 \longrightarrow$ " " $=$ either 1 or 2

$$E_1 = \mathrm{Ker}\begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[\text{operations}]{\text{row}} \mathrm{Ker}\begin{bmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} ay \\ 0 \\ z \end{bmatrix}$$

if $a = 0 \Rightarrow E_1 = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \longrightarrow$ 2 dim $\longrightarrow$ geom mult. $= 2$

if $a \neq 0 \Rightarrow E_1 = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} \longrightarrow$ 1 dim $\longrightarrow$ " " $= 1$

$*$ if we need an eigenbasis to exist for this 3x3 matrix, the dimensions must add up to $\underline{\underline{3}}$

$\Rightarrow a=0$ gives an eigenbasis b/c here,

E_1 is 2-dim and E_2 is 1-dim

Prob 36

trace 1 $\neq$ trace 2 $\Rightarrow$ not similar

Sec 7.4

Prob 8

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \Rightarrow (1-\lambda)^2 - 9 = 0 \Rightarrow \lambda = 4, -2$$

$$E_4 = \text{Ker} \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} = \text{span} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ — ~~and~~ one-dim

$$E_{-2} = \text{Ker} \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} = \text{span} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$ — one-dim

They add up to 2 dimensions and A is 2×2 matrix

$\Rightarrow A$ is diagonalizable

$$S = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad S^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\therefore D = \underbrace{\frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}}_{S^{-1}} \underbrace{\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}}_{S} = \begin{pmatrix} -2 & 0 \\ 0 & 4 \end{pmatrix}$$

the eigenvalues on the diagonal

correct

Prob 16

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \longrightarrow (1-\lambda)(\lambda-2)(\lambda-3) = 0$$

$$\Rightarrow \lambda = 1, 2, 3$$

Distinct eigenvalues $\longrightarrow$ A is diagonalizable

and we expect D to be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Let's find S,

$$E_1 = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \text{span} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 2 & 0 & -2 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \text{Span} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix} = \text{span} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\therefore D = S^{-1} A S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$