HW13

Sec 7.1

Prob 10

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} a+2c \\ b+2d \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \Rightarrow \begin{matrix} a = 5-2c \\ b = 10-2d \end{matrix}$$

$\therefore$ the solution is $\begin{bmatrix} 5-2c & c \\ 10-2d & d \end{bmatrix}$

Prob 16

Any vector will stay as it is but pointing in opposite direction $\therefore T\underline{A} = -\underline{A}$ eigenvalue is -1

Prob 18

* vectors in the reflection plane stay the same

$\rightarrow$ eigen value $= 1$, basis contain two vectors (plane)

* vectors normal to the reflection plane stay the same also but with opposite direction $\rightarrow$ eigenvalue $= -1$

* Any "tilted" vector is not an eigen vector of this transformation.

Prob 34

$A\underline{v} = 4\underline{v} \Rightarrow A \cdot A\underline{v} = A\,4\underline{v} = 16\underline{v}$

$\therefore (A^2 + 2A + 3I_n)\underline{v} = (16 + 2\cdot4 + 3\cdot1)\underline{v} = 27\underline{v}$

It's an eigenvector with $\underline{27}$ as eigenvalue

Prob 50

~~$h(t+1) = A($~~ $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$

(a) $h(0) = f(0) = 100$

eigen vector

$$A\begin{pmatrix}100\\100\end{pmatrix} = \begin{pmatrix}4 & -2\\1 & 1\end{pmatrix}\begin{pmatrix}100\\100\end{pmatrix} = \begin{pmatrix}200\\200\end{pmatrix} = 2\begin{pmatrix}100\\100\end{pmatrix}$$

So, Applying A once to this initial condition $\begin{pmatrix}100\\100\end{pmatrix}$ gives back the same vector with a factor of 2

$\Rightarrow$ Applying A t times gives back $\begin{pmatrix}100\\100\end{pmatrix}$ with a factor of 2^t

$$\therefore \begin{pmatrix}h(t)\\f(t)\end{pmatrix} = 2^t\begin{pmatrix}100\\100\end{pmatrix}$$

(b) $h(0) = 200 \quad f(0) = 100$

eigen vector

$$A\begin{pmatrix}200\\100\end{pmatrix} = \begin{pmatrix}4 & -2\\1 & 1\end{pmatrix}\begin{pmatrix}200\\100\end{pmatrix} = \begin{pmatrix}600\\300\end{pmatrix} = 3\begin{pmatrix}200\\100\end{pmatrix}$$

∴ similar to ⓐ ∴ $\begin{pmatrix} h(t) \\ f(t) \end{pmatrix} = 3^t \begin{pmatrix} 200 \\ 100 \end{pmatrix}$

ⓒ

$$A\begin{pmatrix} 600 \\ 500 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 600 \\ 500 \end{pmatrix} = \begin{pmatrix} 1400 \\ 1100 \end{pmatrix}$$, not a multiple of $\begin{pmatrix} 600 \\ 500 \end{pmatrix}$

but, we can factorize the initial condition in terms of the eigenvalues found in ⓐ, ⓑ

$$A\begin{pmatrix} 600 \\ 500 \end{pmatrix} = A\left[\begin{pmatrix} 200 \\ 100 \end{pmatrix} + 4\begin{pmatrix} 100 \\ 100 \end{pmatrix}\right] \xrightarrow[ⓐ,ⓑ]{\text{using}} = 3\begin{pmatrix} 200 \\ 100 \end{pmatrix} + 4\cdot 2\begin{pmatrix} 100 \\ 100 \end{pmatrix}$$

And after t time,

$$\begin{pmatrix} h(t) \\ f(t) \end{pmatrix} = \begin{pmatrix} 400\, 2^t + 200\, 3^t \\ 500\, 2^t + 100\, 3^t \end{pmatrix}$$

Sec 7.2

Prob 10

$$\text{Det}\begin{bmatrix} -3-\lambda & 0 & 4 \\ 0 & -1-\lambda & 0 \\ -2 & 7 & 3-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (-3-\lambda)(-1-\lambda)(3-\lambda) + 4(2(-1-\lambda)) = 0$$

$$\Rightarrow (-1-\lambda)(-9+\lambda^2+8) = 0 \Rightarrow \lambda = -1 \quad , \lambda = \pm 1$$

∴ eigenvalue $+1$ Algebric multiplicity one

" -1 " " two

Prob 22

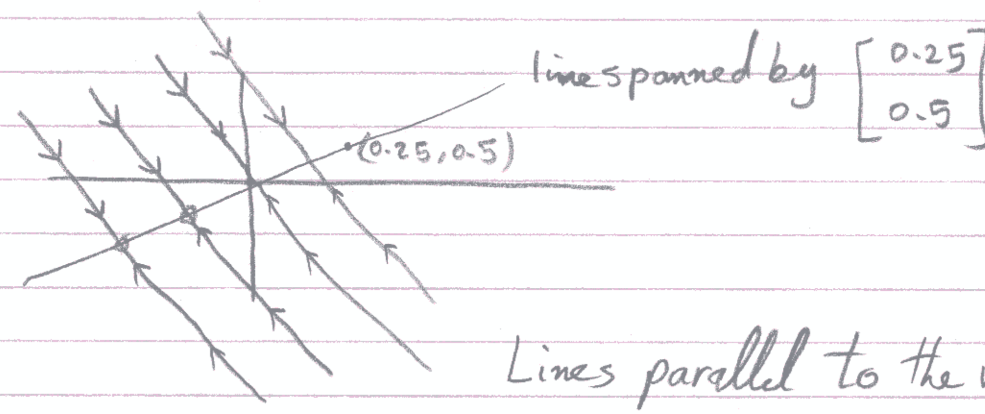
A, A^T have the same eigenvalues
Also " " " trace

$\Rightarrow$ the characteristic polynomial is the same

Prob 24

$\lambda = 1 \ , \ 1/4$

Prob 26



Lines parallel to the vector $(1,-1)$