Problem 1 Numbers in circles refer to eqns.

$$\left\{\begin{array}{l} x+3y+2z=8 \\ x+3y+3z=10 \\ x+4y+2z=9 \end{array}\right\} -2 \longrightarrow \left\{\begin{array}{l} x+3y+2z=8 \\ x+3y+3z=10 \\ 0+y-z=-1 \end{array}\right\} -1 \longrightarrow$$

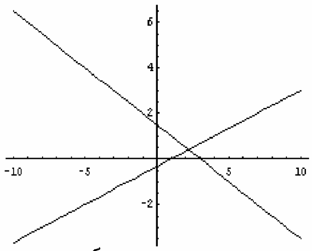
$$\left\{\begin{array}{l} x+3y+2z=8 \\ 0+0+z=2 \\ 0+y-z=-1 \end{array}\right\} \text{swap} \longrightarrow \left\{\begin{array}{l} x+3y+2z=8 \\ 0+y-z=-1 \\ 0+0+z=2 \end{array}\right\} +3 \longrightarrow$$

$$\left\{\begin{array}{l} x+3y+2z=8 \\ 0+y+0=1 \\ 0+0+z=2 \end{array}\right\} -3\times 2 \longrightarrow \left\{\begin{array}{l} x+0+2z=5 \\ 0+y+0=1 \\ 0+0+z=2 \end{array}\right\} -2\times 3 \longrightarrow$$

$$\left\{\begin{array}{l} x=1 \\ y=1 \\ z=2 \end{array}\right\}$$

Problem 2

$$\left\{\begin{aligned} x+2y&=3 \\ x-3y&=1 \end{aligned}\right\} \begin{matrix} \\ -① \end{matrix} \longrightarrow \left\{\begin{aligned} x+2y&=3 \\ 0-5y&=-2 \end{aligned}\right\} \begin{matrix} +\frac{2}{5}\times② \\ \end{matrix} \longrightarrow \left\{\begin{aligned} x&=11/5 \\ y&=2/5 \end{aligned}\right\}$$



Problem 3

$$\left\{\begin{aligned} x+2y+3z&=1 \\ 3x+2y+z&=1 \\ 2x+2y+Kz&=1 \end{aligned}\right\} \begin{matrix} \\ -3\times① \\ -2\times① \end{matrix} \longrightarrow \left\{\begin{matrix} 1 & 2 & 3 & 1 \\ 0 & -4 & -8 & -2 \\ 0 & -2 & K-6 & -1 \end{matrix}\right\} \begin{matrix} \\ \div(-2) \\ \\ \end{matrix}$$

$$\left\{\begin{matrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & 1 \\ 0 & -2 & K-6 & -1 \end{matrix}\right\} \begin{matrix} \\ \\ +② \end{matrix} \longrightarrow \left\{\begin{matrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & K-2 & 0 \end{matrix}\right\}$$

Now, the 3rd eqn is $\boxed{(K-2)z=0}$. If $z=0$, we may continue and solve 2nd eqn $\Rightarrow y=1/2$. Then solve 1st eqn $\Rightarrow x=0$

$\Rightarrow (0, 1/2, 0)$ is a solution [independent of K!]

$\Downarrow$

Part (c) in the problem has the answer [NO]

For any K, the 3 planes intersect in a point (at least)

★ If $K=2$, only the 1st and 2nd eqns survive $\Longrightarrow$ two eqns. in three variables

$\Downarrow$

infinite number of solutions

take $z=t \Rightarrow y=\frac{1-4t}{2}$, substitute in ① $\Rightarrow x+2\left(\frac{1-4t}{2}\right)+3t=1$

$$\Rightarrow x=t$$

$\Rightarrow$ the general solution is $\left(t, \frac{1-4t}{2}, t\right)$ Note: the point $(0, \frac{1}{2}, 0)$ is on this line.

Problem 4

In a straight forward mannar, you can form the system,

$$\begin{cases} 2 = a + b + c \\ 4 = a - b + c & -① \\ 4 = 4a + 2b + c & -4\times① \end{cases} \longrightarrow \begin{Bmatrix} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & -2 & -3 & -4 \end{Bmatrix} \begin{matrix} \\ \div(-2) \\ -② \end{matrix} \longrightarrow$$

$$\begin{Bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -3 & -6 \end{Bmatrix} \begin{matrix} \\ \\ \div(-3) \end{matrix} \longrightarrow \begin{Bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{Bmatrix} \begin{matrix} -② \\ \\ \\ \end{matrix} \longrightarrow$$

$$\begin{Bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{Bmatrix} \begin{matrix} -③ \\ \\ \\ \end{matrix} \longrightarrow \begin{Bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{Bmatrix}$$

$\therefore a=1 \quad , b=-1 \ , \ c=2$

$\therefore f(t)=t^2-t+2 \longrightarrow$ The polynomial required.

Problem 5

You can directly write down these two eqns,

$$\begin{cases} x + 5y + 10z = 100 \\ x + y + z = 22 \end{cases}$$

Two eqns in three variables $\longrightarrow$ Infinite number of solutions (if any)

$\therefore$ let $z = t$; a parameter . Then solve the system ,

$$\begin{cases} x + 5y = 100 - 10t \\ x + y = 22 - t \end{cases} - ② \longrightarrow \begin{cases} 0 + 4y = 78 - 9t \\ x + y = 22 - t \end{cases} \div 4 \longrightarrow$$

$$\begin{cases} y = \frac{78-9t}{4} \\ x + y = 22 - t \end{cases} - ① \longrightarrow \begin{cases} y = \frac{78-9t}{4} \\ x = 22 - t - \frac{78}{4} + \frac{9}{4}t \end{cases}$$

$$\Rightarrow (x, y, z) = \left(\frac{10}{4} + \frac{5}{4}t ,\ \frac{78-9t}{4} ,\ t \right)$$

This infinite number of solutions is filtered using the fact that x, y, z are all integers .

$\Rightarrow$ Try $t = 1 \longrightarrow x, y$ are fractions $\longrightarrow$ not acceptable

$\boxed{t = 2} \longrightarrow x = 5,\ y = 15 \longrightarrow (5, 15, 2)$ is acceptable solution

$t = 3 \longrightarrow x, y$ are fractions $\longrightarrow$ not acceptable

$t = 4$ 〃 〃

$t = 5$ 〃 〃

$\boxed{t = 6} \longrightarrow x = 10,\ y = 6 \longrightarrow (10, 6, 6)$ is the second accepted solution

There is a constraint on the number of solutions because x, y, z are positive numbers,

if $x + 5y + 10z = 100 \Rightarrow \underbrace{(x+y+z)}_{22 \text{ (from eqn. 2)}} + 4y + 9z = 100$

$\Rightarrow 4y + 9z = 78$

$z = 9$ would violate this eqn for positive $y \Rightarrow z \leqslant 8$