Math 211.01: Extra homework

due April 19, 2006

Name: **School ID:**

Answer all the following questions, justifying all your statements. This is an easier version of Midterm 2.

Problem 1. (20 points) (i) Find a basis for the subspace $V := \{(x, y, z) \in \mathbb{R}^3 : x - 2y + 3z = 0\}$.
(ii) Find the coordinates of the vector $\vec{x} = (1, -1, -1)$ with respect to this basis.

Problem 2. (20 points) Let $V = \mathbb{R}^{2\times 2}$. the 2×2 matrices. Which of the following transformations $T : V \to V$ are linear? Explain your answer. If T is linear, describe its image and rank.

(i) $T(M) = M - I$ where $I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(ii) $T(M) = MJ$ where $J := \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$.

Problem 3. (20 points) Let $V = P_2$ the space of polynomials of degree ≤ 2. Define $T : V \to V$ by $T(f) = (1 - t)f'(t)$. Consider the standard basis $\mathcal{B} := (1, t, t^2)$.
(i) What is $T(t^2)$?
(ii) Find the matrix that represents T with respect to the basis $\mathcal{B}$.

Problem 4. (20 points) (i) What is an orthogonal matrix?

(ii) Find constants c, k so that the following matrix is orthogonal.

$$A = \begin{bmatrix} -1/\sqrt{5} & c \\ 2/\sqrt{5} & k \end{bmatrix}$$

Problem 5. (20 points) Let V be the subspace of $\mathbb{R}^3$ spanned by the vectors $\vec{v}_1 = (2, 1, 3)$ and $\vec{v}_2 = (1, 1, -1)$.

(i) Check that $\vec{v}_1, \vec{v}_2$ are orthogonal.

(ii) Find an orthonormal basis for V.

(ii) Find the orthogonal projection $\vec{y}$ of $\vec{x} := (0, 1, 1)$ onto V.

(iii) Check that $\|\vec{x}\|^2 = \|\vec{y}\|^2 + \|\vec{x} - \vec{y}\|^2$.