

Math 211.01: Second Midterm

March 29, 2006

Name:

School ID:

Answer all the following questions, justifying all your statements. Write neatly so that we can read and follow your answers. You are not allowed to use calculators, and please turn off cell phones. Use the back of the exam for scrap. There are five questions. Good luck!

Problem 1. (20 points) Let $V = \mathbb{R}^{2 \times 2}$. the 2×2 matrices. Which of the following transformations $T : V \rightarrow V$ are linear? Explain your answer. If T is linear, describe its image and rank.

(i) $T(M) = M^2$.

(ii) $T(M) = JM - MJ$, where $J := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

1	20pt	
2	20pt	
3	20pt	
4	20pt	
5	20pt	
Total	100pt	

Problem 2. (20 points) (i) Find a basis for the subspace $V := \{(x, y, z, t) \in \mathbb{R}^4 : x - y + 2z + t = 0\}$.

(ii) Find the coordinates of the vector $\vec{x} = (1, 1, -1, 2)$ with respect to this basis.

Problem 3. (20 points) Let $V = P_2$ the space of polynomials of degree ≤ 2 . Define $T : V \rightarrow V$ by $T(f) = (1+t)f'(t)$. Consider the basis $\mathcal{B} := (1+t, t, 1+t^2)$.

(i) Find the coordinates $[f]_{\mathcal{B}}$ of $f(t) = a + bt + ct^2$ with respect to this basis.

(ii) Check that the coordinates of $f = 1+t$ with respect to this basis are $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Why is this so?

(iii) Find the matrix that represents T with respect to this basis.

Problem 4. (20 points) (i) What is an orthogonal matrix?

(ii) Are there constants c, k such that the following matrix is orthogonal? Justify your answer (and calculate c, k if they exist.)

$$A = \begin{bmatrix} 1/\sqrt{3} & 1\sqrt{3} & -1\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & c & k \end{bmatrix}$$

Problem 5. (20 points) Let V be the subspace of \mathbb{R}^4 spanned by the vectors $\vec{v}_1 = (1, 0, 1, 2)$ and $\vec{v}_2 = (1, 1, 1, -1)$.

(i) Check that \vec{v}_1, \vec{v}_2 are orthogonal.

(ii) Find an orthonormal basis for V .

(iii) Find the orthogonal projection \vec{y} of $\vec{x} := (0, 0, 1, 1)$ onto V .

(iv) Check that $\|\vec{x}\|^2 = \|\vec{y}\|^2 + \|\vec{x} - \vec{y}\|^2$.