3.4

(8) No

(10) Yea $3v_1 + v_2$

$[X]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(14) Yea $3v_1 + 4v_2 + 6v_3$

$[X]_B = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

CH3 T/F

(4) No

(14) Yea

(16) No , for example consider the set of vectors that form a line passing through the origin. Any line will be a subspace of the plane $\mathbb{R}^2$ [Do not choose the X axis or Y axis as your line, as they contain $\underline{e}_1, \underline{e}_2$]

Also, you might consider any plane (2D) that contains the origin. Choose a tilted plane that contains none of $\underline{e}_1, \underline{e}_2, \underline{e}_3$.

Now, this plane is a legitimate subspace of the 3D space $\mathbb{R}^3$

Sec 4.1

(4) $\int_0^1 P(t)\, dt = 0$

It'is ~~a~~ subset of P_2 $\therefore P(t) = at^2 + bt + c$

It is a subspace since,

1) $\int_0^1 (0)\, dt = 0$

2) $\int_0^1 K\, P(t)\, dt = K \int_0^1 P(t)\, dt = 0$

3) $\int_0^1 (P_1 + P_2)\, dt = \int_0^1 P_1\, dt + \int_0^2 P_2\, dt = 0 + 0 = 0$

The basis of any element in P_2 is $\{1, t, t^2\}$. the vanishing integration

Puts a relation on the coefficients a, b, c.

$$\int_0^1 (at^2+bt+c)\,dt = \left(\frac{at^3}{3}+\frac{bt^2}{2}+ct\right)\Big|_0^1 = \frac{a}{3}+\frac{b}{2}+c-0 = 0$$

$$\Updownarrow$$

$$\boxed{c = -\left(\frac{a}{3}+\frac{b}{2}\right)}$$

∴ The subspace contains elements on the form,

$$P(t) = at^2+bt-\left(\frac{a}{3}+\frac{b}{2}\right) \Rightarrow P(t) = a\overbrace{\begin{bmatrix} 1 \\ 0 \\ -\frac{1}{3} \end{bmatrix}}^{t^2-\frac{1}{3}} + b\overbrace{\begin{bmatrix} 0 \\ 1 \\ -\frac{1}{2} \end{bmatrix}}^{t-\frac{1}{2}}$$

basis

⑥ Not a subspace,

Consider the matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ that is invertible.

You can find another matrix on the form,

$$B = \begin{bmatrix} -a & -b & -c \\ j & k & l \\ m & n & o \end{bmatrix}$$ also invertible

Try adding both, $A+B = \begin{bmatrix} 0 & 0 & 0 \\ & & \\ & & \end{bmatrix} \Rightarrow$ Non invertible

$\Rightarrow$ Not closed under addition $\Rightarrow$ Not a subspace

(20) $a+d=0 \Rightarrow a=-d$

$$\Rightarrow A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Basis

Dim = 3

(30)

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Longrightarrow \begin{bmatrix} a+2c & b+2d \\ 3a+6c & 3b+6d \end{bmatrix} = 0$

$\Rightarrow \boxed{a=-2c}$, $\boxed{b=-2d}$

$$\therefore A = \begin{bmatrix} -2c & -2d \\ c & d \end{bmatrix} = c\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix}$$

basis

Dim = 2