HW3

Ex 1.3

Prob 6

There are no solutions for this system. Geometrically, we can't construct a combination of $\underline{v}_1, \underline{v}_2$ that gives $\underline{v}_3$

(1)

if $\underline{v}_1 \parallel \underline{v}_2 \Rightarrow \underline{v}_1 = n\underline{v}_2$, $n \in \mathbb{R}$

$$\Rightarrow x\underline{v}_1 + y\underline{v}_2 = nx\underline{v}_2 + y\underline{v}_2 = (nx+y)\underline{v}_2 \neq \underline{v}_3$$

Prob 14

(1)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = -\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \quad \text{Def 1.3.6}$$

$$= \begin{bmatrix} [1 \ 2 \ 3]\cdot[-1 \ 2 \ 1] \\ [2 \ 3 \ 4]\cdot[-1 \ 2 \ 1] \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \quad \text{Fact 1.3.8}$$

Prob 20

(1)

(a) $$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ 3 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 7 & 6 \\ 6 & 6 \end{bmatrix}$$

(b) $$9\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 9 & -9 & 18 \\ 27 & 36 & 45 \end{bmatrix}$$

Prob 30

Assume $A \equiv \begin{pmatrix} a & b & c \\ ka & kb & kc \\ la & lb & lc \end{pmatrix}$ a matrix of rank 1. $\Rightarrow A\begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow \boxed{k=0}$, $(5a+3b-9c)=(2)$, $l(5a+3b-9c)=1$ $\rightarrow \boxed{l = 1/2}$

now, choose any a, b, c such that, $5a+3b-9c=2$

i.e.) $a=1$ $b=1$ $c=2/3$

Prob 36

Assume $A = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$ 3

$$A\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$\therefore x_1 = 1, \quad y_1 = 2, \quad z_1 = 3$

Similarly, we can find all unknowns,

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Prob 49

3

(a) 4 variables 3 eqns $A = 3 \times 4$

if rank $A_{arg} = 2 \longrightarrow$ rank A is two or one

if one $\longrightarrow$ inconsistent system since we have $\left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 1 \end{array}\right]$

if two $\longrightarrow$

# of free variables $= m - \text{rank } A$

$= 4 - 2 = 2 \longrightarrow$ infinite no. of solutions

(b) 3 variables, 4 equations $A = 4 \times 3$

rank $A = 3$

# of free variables $= m - \text{rank } A$

$= 3 - 3 = 0$

$\therefore$ the system has either unique or no solutions [Depending on rank A_{arg}]

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 \end{array}\right]$ unique or, $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{array}\right]$ no solutions

(C) 4 eqns 3 variables Rank $A_{arg} = 4$

$$A_{arg} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{array}\right] \longrightarrow$$ inconsistent system with no solutions

(D) 3 eqns 4 Unknowns Rank $A = 3$

# of free variables $= 4 - 3 = 1 \longrightarrow$ infinite no of solutions

True Or False 20 (1)

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow$ substitute in the system, and you'll get

$c+d=2$ and $c+d=\frac{1}{2}$

inconsistency!

Also,

$$A\begin{bmatrix}2\\2\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix} \Longrightarrow 2A\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix} \Rightarrow A\begin{bmatrix}1\\1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}1\\ \frac{1}{2}\end{bmatrix}$$

but $A\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$ inconsistent

$\therefore$ The statement is incorrect

Ex. 2.1

6 To test the linearity, (3)

$*$ let $\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}v_1+w_1\\v_2+w_2\end{bmatrix}$

$$\therefore T\begin{bmatrix}x_1\\x_2\end{bmatrix}=T\begin{bmatrix}v_1+w_1\\v_2+w_2\end{bmatrix}=(v_1+w_1)\begin{bmatrix}1\\2\\3\end{bmatrix}+(v_2+w_2)\begin{bmatrix}4\\5\\6\end{bmatrix}$$

$$=v_1\begin{bmatrix}1\\2\\3\end{bmatrix}+v_2\begin{bmatrix}4\\5\\6\end{bmatrix}+w_1\begin{bmatrix}1\\2\\3\end{bmatrix}+w_2\begin{bmatrix}4\\5\\6\end{bmatrix}$$

$$=T\begin{bmatrix}v_1\\v_2\end{bmatrix}+T\begin{bmatrix}w_1\\w_2\end{bmatrix}$$

$*$ Also, let $\begin{bmatrix}x_1\\x_2\end{bmatrix}=k\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}ku_1\\ku_2\end{bmatrix}$

$$\therefore Tk\begin{bmatrix}u_1\\u_2\end{bmatrix}=T\begin{bmatrix}ku_1\\ku_2\end{bmatrix}=ku_1\begin{bmatrix}1\\2\\3\end{bmatrix}+ku_2\begin{bmatrix}4\\5\\6\end{bmatrix}$$

$$=k\left[u_1\begin{bmatrix}1\\2\\3\end{bmatrix}+u_2\begin{bmatrix}4\\5\\6\end{bmatrix}\right]=kT\begin{bmatrix}u_1\\u_2\end{bmatrix}$$

$\longrightarrow$ T is linear

Finding the Transformation matrix,

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} x_1+4x_2 \\ 2x_1+5x_2 \\ 3x_1+6x_2 \end{bmatrix}$$

$3\times2 \quad 2\times1 \qquad\qquad 3\times1$

$\therefore$ T is 3×2 matrix

let $T = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \Rightarrow T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ax_1+bx_2 \\ cx_1+dx_2 \\ ex_1+fx_2 \end{bmatrix}$

$T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \longleftarrow$ compare

1

10 let $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$ then solve the system for x, y

$\Rightarrow \begin{aligned} x &= 9a-2b \\ y &= -4a+b \end{aligned} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow A^{-1} \equiv \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$

Another solution,

$\begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix} \longrightarrow \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 4 & 9 & 0 & 1 \end{array}\right]_{-4\times①} \longrightarrow \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -4 & 1 \end{array}\right]^{-2\times②} \longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & 9 & -2 \\ 0 & 1 & -4 & 1 \end{array}\right]$ with the last block $\underbrace{\quad}_{A^{-1}}$

14

(a) $2K-15=0 \Rightarrow K=7.5$ Matrix is invertible if $K \in \mathbb{R}-\{7.5\}$

3

(b) $\begin{bmatrix} 2 & 3 \\ 5 & K \end{bmatrix}^{-1} = \dfrac{1}{2K-15}\begin{bmatrix} K & -3 \\ -5 & 2 \end{bmatrix}$

$2K-15$ is ±1 or a fraction,

$2K-15=\pm1 \Rightarrow 2K=15\pm1 \Rightarrow K=8$ or 7

The only valid fractions are, $\dfrac{K}{2K-15}$ = integer $= n \Rightarrow K = 2Kn-15n \Rightarrow$

$K = \dfrac{15n}{2n-1} \Rightarrow n = \dfrac{K}{2K-15}$