Graded Problems :-

Ex 1.1 — 20

Ex 1.2 — 10 , 22 , 46 ,

And the additional problem.

Ex 1.1

20

From the graph, you can write,

$b = 0.2a + 780$
, $a = 0.1b + 1000$ $\Rightarrow$ $\boxed{a = 1100 \;,\; b = 1000}$ $\rightarrow$ in millions of \$

Ex. 1.2

6

let $x_5 = t$ $\Rightarrow$ $x_4 = 1 - t$

$x_3 = 2 + 2t$

let $x_2 = s$ $\Rightarrow$ $x_1 = 3 - t + 7s$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 - t + 7s \\ s \\ 2 + 2t \\ 1 - t \\ t \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

10

$$\begin{bmatrix} 4 & 3 & 2 & -1 & 4 \\ 5 & 4 & 3 & -1 & 4 \\ -2 & -2 & -1 & 2 & -3 \\ 11 & 6 & 4 & 1 & 11 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} & \frac{-1}{4} & 1 \\ 5 & 4 & 3 & -1 & 4 \\ -2 & 2 & -1 & 2 & -3 \\ 11 & 6 & 4 & 1 & 11 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} & \frac{-1}{4} & 1 \\ 0 & 1 & 2 & 1 & -4 \\ 0 & -\frac{1}{2} & 0 & \frac{3}{2} & -1 \\ 0 & \frac{-9}{4} & \frac{-3}{2} & \frac{15}{4} & 0 \end{bmatrix} \longleftarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} & \frac{-1}{4} & 1 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & -1 \\ 0 & -\frac{1}{2} & 0 & \frac{3}{2} & -1 \\ 0 & -\frac{9}{4} & \frac{-3}{2} & \frac{15}{4} & 0 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 4 \\ 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 3 & 6 & -9 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 & -1 & 4 \\ 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 2 & -3 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -3 & 2 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

let $x_4 = t \implies x_3 = -3 - 2t$

$x_2 = 2 + 3t$

$x_1 = 1 - t$

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b, c, d are in the reduced row-echelon form

22

Seven!

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & k \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & k & l \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & k \\ 0 & 1 & l \end{bmatrix}, \begin{bmatrix} 1 & k & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

k, l are arbitrary numbers

26 Yes

34

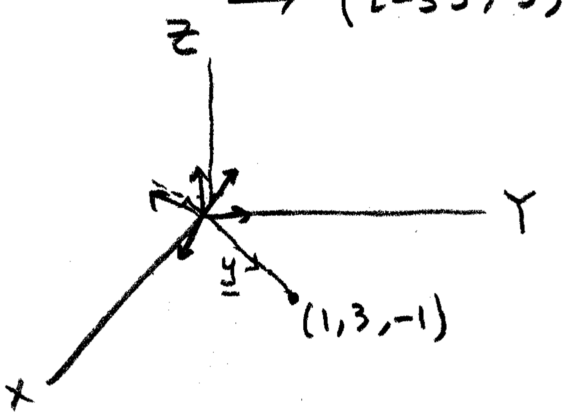
$\underline{x} \cdot \underline{y} = 0$ let $\underline{x} = (x, y, z)$, $\underline{y} = (1, 3, -1)$

$\Rightarrow \underline{x} \cdot \underline{y} = x + 3y - z = 0$ (if perpendicular)

solve the system,

let $z = t$, $y = s$

$\Longrightarrow (t-3s, s, t)$ represent the set of all normal vectors.



(36)

Form the matrix,

$$\begin{array}{c} \begin{bmatrix} 1 & 2 & 4 & -8 \\ 4 & 5 & 6 & -1 \\ 7 & 8 & 9 & 2 \\ 5 & 3 & 1 & 15 \end{bmatrix} \end{array} \begin{array}{l} \\ -4\times 1 \\ -7\times 1 \\ -5\times 1 \end{array} \longrightarrow \begin{bmatrix} 1 & 2 & 4 & -8 \\ 0 & -3 & -10 & 31 \\ 0 & -6 & -19 & 58 \\ 0 & -7 & -19 & 55 \end{bmatrix} \begin{array}{l} \\ \\ -4 \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & 4 & -8 \\ 0 & -3 & -10 & 31 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -19 & 76 \end{bmatrix} \longleftarrow \begin{bmatrix} 1 & 2 & 4 & -8 \\ 0 & -3 & -10 & 31 \\ 0 & 1 & 0 & 3 \\ 0 & -7 & -19 & 55 \end{bmatrix} \begin{array}{l} \\ \\ \\ +7\times 3 \end{array}$$

$$\Big\downarrow \; 4 \div -19$$

$$\begin{bmatrix} 1 & 2 & 4 & -8 \\ 0 & -3 & -10 & 31 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{bmatrix} \begin{array}{l} -4\times 4 \\ +10\times 4 \\ \\ \\ \end{array} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 8 \\ 0 & -3 & 0 & -9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{bmatrix} \begin{array}{l} -2\times 3 \\ \div -3 \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

Notice the redundant eqn. (since we have four eqns in three variables)

$\therefore \; (x_1, x_2, x_3) = (2, 3, -4)$

Prob 46 obtaining the row-echelon form,

$$\begin{bmatrix} 0 & 1 & 2k & 0 \\ 1 & 2 & 6 & 2 \\ k & 0 & 2 & 1 \end{bmatrix} \xrightarrow{\text{swap}} \begin{bmatrix} k & 0 & 2 & 1 \\ 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 2/k & 1/k \\ 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2/k & 1/k \\ 0 & 2 & 6-\frac{2}{k} & 2-1/k \\ 0 & 1 & 2k & 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 2/k & 1/k \\ 0 & 1 & 2k & 0 \\ 0 & 2 & 6-\frac{2}{k} & 2-\frac{1}{k} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2/k & 1/k \\ 0 & 1 & 2k & 0 \\ 0 & 0 & 6-\frac{2}{k}-4k & 2-1/k \end{bmatrix}$$

* Consider the third eqn,

1) if it is on the form $0 \quad 0 \quad 0 \quad \text{number}$ → inconsistent system → no solution

2) " " " $0 \quad 0 \quad \text{number} \quad \text{number}$ → there is a unique solution

3) " " " $0 \quad 0 \quad 0 \quad 0$ → z is a free parameter and there is infinite no. of solutions

4) " " " $0 \quad 0 \quad \text{number} \quad 0$ → Unique solution with $z=0$

So, let $6-\frac{2}{k}-4k=0$

$\Rightarrow 3k-1-2k^2=0$

$2k^2-3k+1=0 \Rightarrow (2k-1)(k-1)=0$

$\Rightarrow \boxed{k=+1}$ or $\boxed{k=1/2}$

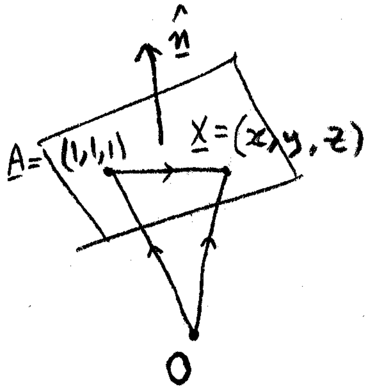
- if $k=1$ → case (1) → no solutions
- if $k=1/2$ → Case (3) → parametric solutions, let $z=t \Rightarrow y=-t$, $x=2-4t$
- Unique solution elsewhere

The written problem,

(a) Assume $z=t$ and solve the system $\longrightarrow$

$$z = t$$
$$y = -1-2t$$
$$x = 2+t$$

(b)



$\hat{n} \equiv$ (the vector normal to the plane)

$= (2,-3,1) \longleftrightarrow$ from the line parametric eqn.

$$\hat{n} \cdot \underbrace{(\underline{X} - \underline{A})}_{\text{vector in the plane}} = 0$$

$$\Rightarrow (2,-3,1) \cdot \big((x,y,z) - (1,1,1)\big) = 0$$

$$\Rightarrow 2x - 2 - 3y + 3 + z - 1 = 0$$

$$\Rightarrow 2x - 3y + z = 0$$