MAT211 - Introduction to Linear Algebra
SUMMER 07

Sample midterm

Question 1. Consider the vectors $\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$.
(a) Find $\vec{v} + \vec{x}$, $\vec{v} - \vec{x}$, $\vec{x} - \vec{v}$, $\text{proj}_L(\vec{x})$ and $\text{ref}_L(\vec{x})$, where $L = \text{span}\{\vec{v}\}$.
(b) Represent graphically the vectors found in (a).

Question 2. Consider a linear system $A\vec{x} = \vec{b}$ where A is a $m \times n$ matrix. You are told that this linear system always has a solution, no matter which $\vec{b}$ you choose.
(a) Can you conclude that A is invertible?
(b) What can you say about the kernel of A?

Question 3. Consider the linear transformation $T : \mathbb{R}^6 \to \mathbb{R}^4$ given by $T(\vec{x}) = A\vec{x}$, where

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

(a) Find bases for the image and the kernel of T.
(b) Can you find three linearly independent vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}$ such that $T(\vec{v_1}) = \vec{0}$, $T(\vec{v_2}) = \vec{0}$, $T(\vec{v_3}) = \vec{0}$?
(c) Does the linear system $A\vec{x} = \vec{b}$ have a solution for every $\vec{b} \in \mathbb{R}^4$? If not, for which vectors $\vec{b} \in \mathbb{R}^4$ does it have a solution?

Question 4. Consider the set of vectors.

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(a) Verify that $\mathcal{B}$ is a basis for $\mathbb{R}^3$.
(b) Write $\vec{x} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ in the basis $\mathcal{B}$, i.e., find the components of $[\vec{x}]_{\mathcal{B}}$.

Question 5. Which of the follwing sets are vector spaces?
(a) the set of all polynomials of degree n such that $p(0) = 0$.
(b) the set of all polynomials p of degree 2 such that $p(1) + p(0) = 0$.
(c) the set of all invertible 2×2 matrices.
(d) the set of all infinite sequences of real number $(x_0, x_1, x_2, \dots)$ such that $\lim_{n\to\infty} x_n = 0$.
(e) the set of all postive funcions on $\mathbb{R}$.
(f) the set of all solutions of the linear system $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.
(g) the set of real numbers $\mathbb{R}$.
(h) the set of all funcions on $\mathbb{R}$ which are not continuous.
(i) the set of all matrices, no matter which size.
(j) the set of all functions $f : [0, 1] \to \mathbb{R}$ which are continuous.
(k) the set $L(\mathbb{R}^m, \mathbb{R}^n)$ of all linear transformations from $\mathbb{R}^m$ to $\mathbb{R}^n$.

Question 6. For the vector spaces of the previous exercise, which ones are finite dimensional and which ones are infinite dimensional? For the finite dimensional ones, give their dimensions.