

Math 127 - Spring 2008

Practice for First Examination

1. Calculate the following limit if it exists.

$$\lim_{n \rightarrow \infty} \frac{e^{2n} + n^5 e^{5n}}{n^4(3ne^{2n} + 1)(4e^{3n} + 5n)}.$$

2. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{1 + \ln(n)}{n} e^{-(1+\ln(n))^2}$$

is convergent or divergent. Justify your answer.

3. Calculate the following limit if it exists.

$$\lim_{n \rightarrow \infty} e^{(2/n) - n^2}.$$

4. Decide whether the series

$$\sum_{n=1}^{\infty} \ln \left(1 - \frac{1}{n^2} \right)$$

converges or diverges. If it converges, calculate the limit exactly. If it diverges, explain why.
(Hint: $1 - \frac{1}{n^2} = \frac{n^2-1}{n^2} = \frac{(n-1)(n+1)}{n \cdot n}$ and $\ln(ab) = \ln a + \ln b$.)

5. Does the series

$$\sum_{j=1}^{\infty} \frac{(-1)^j}{j \ln(j)}$$

converge or diverge?

6. Find a number N such that

$$\left(\sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}} \right) - \left(\sum_{n=1}^N \frac{n^2}{e^{n^3}} \right) < 10^{-6}.$$