Answers for HW 9

3. Usually, one may compute the 1st, 2nd and 3rd derivatives of f at $\frac{\pi}{6}$ to get the T_3. For this problem, one may also use $\sin(y+z) = \sin y \cos z + \cos y \sin z$ to get the whole Taylor series of f. Just take $y = x - \frac{\pi}{6}$ and $z = \frac{\pi}{6}$.(Since we know how to express $\sin t$ or $\cos t$.)

5. Except getting the derivatives of f, one may employ 9(b) of section 8.8 to get the whole Taylor series.

17. Since

$$x \sin x = \sum_{k=1}^{\infty} (-1)^{(n+1)} \frac{x^{2n}}{(2n-1)!}$$

we can get the whole Taylor series. One may use

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}.$$

Since $(x \sin x)^{(5)} = 5 \sin x + x \cos x$, roughly we can get an estimation for this question if we take $M = 6$ (a little better one is $\sqrt{26}$ because of $5 \sin x + x \cos x = \sqrt{5^2 + x^2} \sin(x + \arctan \frac{x}{5})$) Another way is the Alternating Series Estimation Theorem (for each fixed x on [-1,1]). The error should be less than $\frac{x^6}{5!} \leq \frac{1}{5!}$.

19. By ex(3) and the fact $|(\sin x)^{(4)}| = |\sin x| \leqslant 1$, $|R_3| \leqslant \frac{1}{4!}(\frac{5 \times 2\pi}{360})^4 \approx 2.4 \times 10^{-6}$. Since the third item is about -9.6×10^{-5}, we must take $T_3(35°)$.

23. By Alternating Series Estimation Theorem, we just need to determine on what interval the following holds:

$$|\frac{x^5}{5!}| \leq 0.01.$$

By Taylor's Inequality, it is to find: (if let $M = 1$ roughly)

$$\frac{x^4}{4!} \leq 0.01.$$

25. Let this moment time $t = 0$ and set $S(t)$ distance function from time-zero position (hence $S(0) = 0$). The hypothesis tells us $(\frac{d}{dt}S)(0) = 20$ and $(\frac{d^2}{dt^2}S)(0) = 2$, so the Taylor's polynomial is:

$$S(t) = 20t + t^2 + \ higher\ order\ items\ w.r.t\ t.$$

Therefore, we would like to take 21 as an approximation for $S(1)$. No for the second question, since when t is large, we should never easily ignore the higher order items of t.