Answers for HW 3

2. Since f is continuous, positive and decreasing, we have

$$\sum_{1}^{5} a_n \geq \int_{1}^{6} f(x)dx = \sum_{1}^{5} \int_{i}^{i+1} f(x)dx \geq \sum_{2}^{6} a_n.$$

6. $p < 1$, so divergent.

7. $p > 1$, so convergent.

8. In fact, we can use the fact $\frac{1}{n^2+1} < \frac{1}{n^2}$ and apply the Integral Test to the latter one to get the conclusion it is convergent. However, we can also apply Intergral Test directly, since $\int \frac{1}{x^2+1}dx = \arctan x + C$.

9. We can use either $\frac{1}{n^2+n+1} < \frac{1}{n(n+1)}$ or $\frac{1}{n^2+n+1} < \frac{1}{n^2}$ to know its convergence.

10. Divergent by comparing with $\frac{1}{\sqrt{n}}$.

11. It is the case $a_n = \frac{1}{n^3}$ with $p = 3 > 1$. Convergent.

12. Both $\sum a_n \; and \sum b_n$ are convergent, where $a_n = \frac{5}{n^4}$ and $b_n = \frac{4}{n\sqrt{n}}$.

13. Since $\forall$ fixed $q \in R$, we have $lim_{n\to\infty}\frac{n^q}{e^n} = 0$, there exist sufficiently large number M, s.t. $\forall k > M,\ k^q e^{-k} < 1$. Specially, when $q = 3$, it is $ke^{-k} < \frac{1}{k^2}$. So it is convergent. One generalization for The Limit Comparison Test in book is if instead the sum of b_n is convergent and $\lim \frac{a_n}{b_n} < 1$, then the sum of a_n converges too.

14. Divergent. Compare with $\frac{1}{n}$.

15. Divergent. Integral Test for $\int \frac{1}{x \ln x}dx = \ln \ln x + C$. (Assume $x \geq 2$ so that $\ln \ln(x)$ makes sense.)

16. Convergent. Integral Test for $\frac{1}{n^2}$.

34. One can use $\frac{n}{(n+1)3^n} < \frac{1}{3^n}$ to give a rough estimation.