Answers to the MAT127 Homework No.2

Chapter 8 Section 2 Problem 5, 7, 9, 19, 20, 25-28, 33, 36, 43

5. Divergent. Because $\tan n$ does not convergent to 0.
7. Convergent. Since

$$\begin{aligned}&\sum_{n=1}^{N}\left(\frac{1}{n^{1.5}}-\frac{1}{(n+1)^{1.5}}\right)\\ =\ & \left(1-\frac{1}{2^{1.5}}\right)+\left(\frac{1}{2^{1.5}}-\frac{1}{3^{1.5}}\right)+\left(\frac{1}{3^{1.5}}-\frac{1}{4^{1.5}}\right)\\ &+\ldots+\left(\frac{1}{N^{1.5}}-\frac{1}{(N+1)^{1.5}}\right)\\ =\ & 1-\frac{1}{(N+1)^{1.5}},\end{aligned}$$

we have

$$\begin{aligned}&\sum_{n=1}^{\infty}\left(\frac{1}{n^{1.5}}-\frac{1}{(n+1)^{1.5}}\right)\\ =\ & \lim_{N\to\infty}\sum_{n=1}^{N}\left(\frac{1}{n^{1.5}}-\frac{1}{(n+1)^{1.5}}\right)\\ =\ & \lim_{N\to\infty}\left(1-\frac{1}{N^{1.5}}\right)=1.\end{aligned}$$

9. (a) Convergent. (Limit is $\frac{2}{3}$.)
   (b) Divergent, for $a_n \to 2/3 \neq 0$ as $n \to \infty$.
19. Divergent, since $k^2/(k^2-1) \to 1 \neq 0$ as $k \to \infty$.
20. Divergent, for the similar reason of problem 19.
25. Divergent, for $\arctan n \to \pi/2$ as $n \to \infty$
26. $a = \cos 1, \quad |r| = \cos 1 < 1$, so it converges to $\cos 1/(1-\cos 1)$.
27. Since

$$\frac{2}{n^2-1}=\frac{2}{(n-1)(n+1)}=\frac{1}{n-1}-\frac{1}{n+1},$$

$$\begin{aligned}&\sum_{n=2}^{N}\frac{2}{n^2-1}\\ =\ & \left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\ldots+\left(\frac{1}{N-1}-\frac{1}{N+1}\right)\\ =\ & 1+\frac{1}{2}-\frac{1}{N}-\frac{1}{N+1}.\end{aligned}$$

Then

$$\sum_{n=2}^{\infty}\frac{2}{n^2-1}=\lim_{N\to\infty}\sum_{n=2}^{N}\frac{2}{n^2-1}=1+\frac{1}{2}=\frac{3}{2}.$$

28. Since
$$\frac{2}{n^2+4n+3} = \frac{2}{(n+1)(n+3)} = \frac{1}{(n+1)} - \frac{1}{(n+3)},$$
$$\begin{aligned}
&\sum_{n=1}^{N} \frac{2}{n^2+4n+3} \\
= &\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\ldots+\left(\frac{1}{N+1}-\frac{1}{N+3}\right) \\
= &\frac{1}{2}+\frac{1}{3}-\frac{1}{N+2}-\frac{1}{N+3}.
\end{aligned}$$
Then
$$\sum_{n=2}^{\infty} \frac{2}{n^2+4n+3} = \lim_{N\to\infty} \sum_{n=2}^{N} \frac{2}{n^2+3n+4} = \frac{1}{2}+\frac{1}{3} = \frac{5}{6}.$$
33.
$$\begin{aligned}
0.\overline{417} &= 0.417(1+0.001+0.000001+...) = 0.417\sum_{n=0}^{\infty} 10^{-3n} \\
&= 0.417 \cdot \frac{1}{1-10^{-3}} = \frac{417}{999} = \frac{139}{333}.
\end{aligned}$$
So, $3.417 = 3 + \frac{139}{333} = \frac{1138}{333}$.

36.
$$\sum_{n=0}^{\infty} 2^n (x+1)^n = \sum_{n=0}^{\infty} (2(x+1))^n$$
So this is a geometric series with $a = 1$ and $r = 2(x+1)$. The series converges if and only if $r < 1$. Therefore, when $-3/2 < x < -1/2$, the series converges to $1/(1-4(x+1)) = 1/(3-4x)$. For other values of x the series diverges.

43. (a) $S_n = D + cD + c^2 D + ... + c^{n-1} D = D\frac{1-c^n}{1-c}$.

(b)
$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} D\frac{1-c^n}{1-c} = \frac{D}{1-c} = \frac{1}{s}D.$$
When $c = 80\%$, $s = 1-c = 0.2$. Hence $k = 1/s = 5$.