Answers for HW 14

1.
$$\lambda^2 - 13\lambda + 42 = (\lambda - 6)(\lambda - 7),$$
so solutions should be $y(x) = pe^{6x} + qe^{7x}$.

2.
$$\lambda_{1,2} = \frac{-7 \pm \sqrt{7^2 - 4 \times 3}}{2},$$
So solutions should be $y(x) = pe^{\frac{-7+\sqrt{37}}{2}x} + qe^{\frac{-7-\sqrt{37}}{2}x}$.

3.
$$\lambda_{1,2} = \frac{3 \pm \sqrt{3^2 - 4 \times 8}}{2},$$
So we get solutions $y(x) = pe^{\frac{3}{2}x} \cos \sqrt{23}x + qe^{\frac{3}{2}x} \sin \sqrt{23}x$.

4.
$$\lambda_{1,2} = \pm 2\sqrt{3},$$
so $y(x) = pe^{2\sqrt{3}x} + qe^{-2\sqrt{3}x}$.
5.
$$\lambda_{1,2} = \pm 2\sqrt{3}i,$$
so $y(x) = p \cos 2\sqrt{3}x + q \sin 2\sqrt{3}$.
6.
$$\lambda_{1,2} = \frac{3}{2},$$
so $y(x) = pe^{\frac{3}{2}x} + qxe^{\frac{3}{2}x}$.
7.
$$\lambda_{1,2} = 0 \; and \; -\frac{3}{2},$$
so $y(x) = pe^{0x} + qe^{-\frac{3}{2}x} = p + qe^{-\frac{3}{2}x}$.
8.
$$\lambda_{1,2} = 6 \; and \; -5,$$
so $y(x) = pe^{6x} + qe^{-5x}$.
9.
$$\lambda_{1,2} = 2,$$
so $y(x) = pe^{2x} + qxe^{2x}$.
10.
$$\lambda_{1,2} = \frac{2 \pm \sqrt{2^2 - 4 \times 5}}{2 \times 5} = \frac{1 \pm 2i}{5},$$
so $y(x) = pe^{\frac{1}{5}x} \cos \frac{2}{5}x + qe^{\frac{1}{5}x} \sin \frac{2}{5}x$.
11.
$$\lambda_{1,2} = 2 \; and \; 3,$$

so $y(x) = pe^{2x} + qe^{3x}$ *and* $y' = 2pe^{2x} + 3qe^{3x}$.
The initial values mean

$$p + q = 1 \quad and \quad 2p + 3q = 1, \quad i.e.$$

$p = 2$ *and* $q = -1$. Then just plug in those numbers in general solutions above.

12.

$$\lambda_{1,2} = -1,$$

so $y(x) = pe^{-x} + qxe^{-x}$ *and* $y' = -pe^{-x} - qxe^{-x} + qe^{-x}$.
The initial values mean

$$pe^{-2} + 2qe^{-x} = 1 \quad and \quad -pe^{-2} - 2qe^{-2} + qe^{-2} = 2, \quad i.e.$$

$p = -5e^2$ *and* $q = 3e^2$. Then plug in these numbers.

13.

$$\lambda_{1,2} = \pm\frac{i}{2},$$

so $y(x) = p\cos\frac{x}{2} + q\sin\frac{x}{2}$ *and* $y' = -\frac{1}{2}p\sin\frac{x}{2} + \frac{1}{2}q\cos\frac{x}{2}$.
The initial values mean

$$q = 1 \quad and \quad -\frac{1}{2}p = -1, \quad i.e.$$

$p = 2$ *and* $q = 1$. Then plug in these numbers.

14.

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i,$$

so $y(x) = pe^x\cos x + qe^x\sin x$ *and* $y' = pe^x\cos x - pe^x\sin x + qe^x\sin x + qe^x\cos x$.
The initial values mean

$$p = -1 \quad and \quad p + q = -1, \quad i.e.$$

$p = -1$ *and* $q = 0$. Then plug in these numbers.

15.

$$\lambda_{1,2} = -2,$$

so $y(x) = pe^{-2x} + qxe^{-2x}$ *and* $y' = -2pe^{-2x} + qe^{-2x} - 2qxe^{-2x}$.
The initial values mean

$$pe^2 - qe^2 = 2 \quad and \quad -2pe^2 + q^2 + 2qe^2 = 1, \quad i.e.$$

$p = 7e^{-2}$ *and* $q = 5e^{-2}$. Then plug in these numbers.

16.

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i,$$

so $y(x) = pe^x \cos 2x + qe^x \sin 2x$ and $y' = pe^x \cos 2x - 2pe^x \sin 2x + qe^x \sin 2x + 2qe^x \cos 2x$.
The initial values mean

$$-pe^{\frac{\pi}{2}} = 0 \quad and \quad -pe^{\frac{\pi}{2}} - 2qe^{\frac{\pi}{2}} = 2, \quad i.e.$$

$p = 0$ and $q = -e^{-\frac{\pi}{2}}$. Then plug in these numbers.