Answers to the MAT127 Homework No.10

Chapter 7 Section 1 Problem 3-12, Section 2 Problem 2, 3, 5, 7, 9-13, 18, 20

Section 7.1

3.(a) Substitute $y = e^{rx}$ into the equation, we get

$$2(e^{rx})'' + (e^{rx})' - e^{rx} = 0.$$

Hence,

$$2r^2e^{rx} + re^{rx} - e^{rx} = 0.$$

Cancel the e^{rx} in both sides of the equation, we have

$$2r^2 + r - 1 = 0.$$

Solve this equation, we have $r = -1$ or $r = 1/2$.
(b) Substitute y into the differential equation, by straightforward computation, we can verify it is the solution.

4.(a) Substitute $y = \cos kt$ into the differential equation, by direct computation we get

$$-4k^2 \cos kt = -25 \cos kt, \quad \text{i.e.} \quad k^2 = \frac{25}{4}.$$

Then, we have $k = \pm 5/4$.
(b) By direct computation we can verify the fact.

5. By substitute all four choices into the differential equation and by direct computation we get that (c) and (d) are the solutions.
6.(a) Just substitute y into the differential equation and calculate directly.
(b) Omitted.
(c)$2 = y(1) = (\ln 1 + C)/1 = C$, hence $C = 2$. Then $y = (\ln x + 2)/x$ is the solution that we want.
(d)$1 = y(2)(\ln 2 + C)/2$, hence $C = 2 - \ln 2$. Then $y = (\ln x + 2 - \ln 2)/x$ is the solution that we want.

7.(a) Yes. Since $(1/x)' = -(1/x)^2$, $1/x$ is one of the solution.
(b) Just substitute it into the equation and by straightforward computation.

(c) Yes. $y \equiv 0$.
(d) $0.5 = y(0) = 1/(0 + C) = C$, hence $C = 2$. Therefore the solution of the initial problem is $y = 1/(x + 2)$.

8.(a) When x is close to 0, $y' = xy^2$ is close to 0. Hence, the graph of the solutions are closed to the parallel straight lines when x is close to 0.
(b) Just substitute it into the equation and by straightforward computation.
(c) Omitted.
(d)$2 = y(0) = (c - 0)^{-1/2} = 1/\sqrt{c}$, hence $c = 1/4$. Therefore the solution is $y = (1/4 - x^2)^{-1/2}$.
9.(a) The population is increasing iff

$$P' = 1.2P\left(1 - \frac{P}{4200}\right) > 0.$$

Hence, $(1 - P/4200) > 0$ i.e. $0 < P < 4200$.(Note that the population should not be less than zero.)
(b) The population is decreasing iff

$$P' = 1.2P\left(1 - \frac{P}{4200}\right) < 0.$$

Hence, $(1 - P/4200) < 0$ i.e. $P > 4200$.
(c) The equilibrium solution is $P' = 0$. Hence it is $P \equiv 0$ or $P = 4200$.

10.(a) The constant solution must satisfies

$$0 = y' = y^4 - 6y^3 + 5y^2 = y^2(y^2 - 6y + 5) = y^2(y - 1)(y - 5).$$

Therefore the constant solutions are $y \equiv 0$, $y \equiv 1$, and $y \equiv 5$.
(b)y is increasing iff

$$y' = y^2(y - 1)(y - 5) > 0,$$

which is equivalent to $y < 0$, $0 < y < 1$, or $5 < y$.
(c)y is decreasing iff

$$y' = y^2(y - 1)(y - 5) < 0,$$

which is equivalent to $1 < y < 5$.

11.(a) Since $y' \geq 0$, the solution cannot be decreasing. Hence (a) is impossible.

(b) $y'(1) = 0$ but the derivative of the graph at this point is not.

12. Equation A cannot be a solution, because when $x > 0$, from the graph, $y > 0$, hence $y' > 0$. But, in the graph is decreasing when $x > 0$ is large.
Equation B cannot be a solution neither. Because $y'(0) = -2 \cdot 0 \cdot y = 0$, but in the graph, $y'(0) \neq 0$. So, the answer should be equation C.

Section 7.2

Since all problems require depicting graphs, we omit the solutions. If you have questions, please ask your TAs and Instructors.