MAT126 — Calculus B
Solutions to some practice problems

sec 4.9, problem 24. Find f if $f''(x) = 4 - 6x - 40x^3$ and $f(0) = 2$, $f'(0) = 1$.
Computing the antiderivative:

$$f'(x) = \int f''(x)dx = \int (4 - 6x - 40x^3)dx = 4x - 6\frac{x^2}{2} - 40\frac{x^4}{4} + C$$
$$= 4x - 3x^2 - 10x^4 + C$$
$$f'(0) = 1 = C \Rightarrow f'(x) = 4x - 3x^2 - 10x^4 + 1$$

Doing it again:

$$f(x) = \int f'(x)dx = \int (4x - 3x^2 - 10x^4 + 1)dx = 4\frac{x^2}{2} - 3\frac{x^3}{3} - 10\frac{x^5}{5} + x + C$$
$$= 2x^2 - x^3 - 2x^5 + x + C$$
$$f(0) = 2 = C \Rightarrow f(x) = 2x^2 - x^3 - 2x^5 + x + 2$$

sec 5.2, problem 18. Express

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1 + x_i}\Delta x$$

as a definite integral on $[1, 5]$.

Comparing the given expression with

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

we conclude that $f(x) = \frac{e^x}{1+x}$. Since the given interval is $[1, 5]$ we have $a = 1$ and $b = 5$, so:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1 + x_i}\Delta x = \int_1^5 \frac{e^x}{1 + x}dx$$

sec 5.3, problem 10. Evaluate $\int_0^4 (2v + 5)(3v - 1)dv$.

$$\int_0^4 (2v + 5)(3v - 1)dv = \int_0^4 (6v^2 + 13v - 5)dv = 6\frac{v^3}{3}\Big|_0^4 + 13\frac{v^2}{2}\Big|_0^4 - 5v\Big|_0^4 =$$
$$2 \times 4^3 + \frac{13}{2} \times 4^2 - 5 \times 4 = 212$$

sec 5.4, problem 12.Compute the derivative of $h(x) = \int_0^{x^2} \sqrt{1 + r^3}dr$.

Put $u = x^2$, so that h becomes $h(u) = \int_0^u \sqrt{1 + r^3}dr$. By the Fundamental Theorem of Calculus:

$$\frac{dh}{du} = \frac{d}{du}\int_0^u \sqrt{1 + r^3}dr = \sqrt{1 + u^3}$$

But $u = x^2$, so $\frac{dh}{du} = \sqrt{1 + (x^2)^3} = \sqrt{1 + x^6}$. Now we apply the Chain Rule:

$$\frac{dh}{dx} = \frac{dh}{du}\frac{du}{dx} = \sqrt{1 + x^6}\frac{dx^2}{dx} = 2x\sqrt{1 + x^6}$$