

Starting next week, in order to make the class more fun, we will be watching, and doing problems from, a number of entertaining filmstrips written by Martin Gardner. Martin Gardner used to write the Mathematical Games column for Scientific American. Since there won't be much time to discuss homework problems, here are just a couple.

**Problem 1.** *Let us say that a natural number “has a binary expansion” if it can be written as a sum of several distinct powers of 2. For example*

$$8 = 2^3$$

$$10 = 2^3 + 2^1$$

*Show by induction that every natural number has a binary expansion. (Hint: the statement  $P(n)$  is that “every natural number  $\leq n$  has a binary expansion”.)*

*(Aside: this isn't really helpful in solving the problem, but you may have seen this fact before written differently. Namely, we can write the equations above as*

$$8 = 1 * 2^3 + 0 * 2^2 + 0 * 2^1 + 0 * 2^0$$

$$10 = 1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 0 * 2^0$$

*and because of these equations, we say that “in binary, 8 is 1000, and 10 is 1010”. )*

**Problem 2.** *Show by induction that*

$$(1 + x)^n > 1 + nx$$

*whenever  $x > -1$ ,  $x \neq 0$ , and  $n$  is a natural number which is greater than 1.*