

**Problem 1.** (a) Let  $n$  be a positive integer, and let  $m$  be the sum of its digits. Show that, no matter what  $n$  is,  $n - m$  must be divisible by 9.

(b) In (a), show that  $n$  and  $m$  have the same remainders when divided by 9.

(c) In (a), show that  $n$  and  $m$  have the same remainders when divided by 3.

**Problem 2.** Two children take turns breaking up a rectangular chocolate bar 6 squares wide by 8 squares long. They may break the bar only along the divisions between the squares. If the bar breaks into several pieces, they keep breaking the pieces up until only the individual squares remain. The player who cannot make a break loses the game. Who will win?

**Problem 3.** Two children play the following game. There are two piles of 7 stones each. At each turn, a player may take as many stones as she chooses, but only from one of the piles. The winner is the one who takes the last stone. Which player can force a win, and how?