Problem 1. *Let n be a natural number.*
(a) Show that $n(n+1)$ is always divisible by 2.
(b) Show that $n(n+1)(n+2)$ is always divisible by 3.
(c) More generally, suppose that m is another natural number. Show that $n(n+1)\ldots(n+m-1)$ is always divisible by m.

Problem 2. *Suppose p is a prime number, and that $p > 3$.*
(a) What is the remainder if you divide p^2 by 3?
(b) Using (a), prove that $2p^2+1$ cannot be prime.

Problem 3. *Suppose that the natural numbers x, y and z satisfy $x^2+y^2=z^2$. Show that at least one of them must be divisible by 3.*

Problem 4. *In the last homework we showed that if x is any natural number, then x and x^3 have the same remainders when divided by 6. Use this to prove the following:*
Say a, b and c are natural numbers and that $a+b+c$ is divisible by 6. Show that $a^3+b^3+c^3$ is also divisible by 6.

Problem 5. *Given 30 integers, show that two of them can be chosen whose difference is divisible by 29.*