

Problem 1. Ten students solved a total of 35 problems in a math competition. Each problem was solved by exactly one student. There is at least one student who solved exactly one problem, at least one student who solved exactly two problems, and at least one student who solved exactly three problems. Prove that there is also at least one student who solved at least five problems.

Problem 2. The sum of seven given numbers is 332. Prove that three of them can be chosen so that the sum of the numbers is no less than 142.

Problem 3. Given 8 different natural numbers, none greater than 15, show that at least three pairs of them have the same positive difference (the pairs need not be disjoint as sets).

Problem 4. Let p, q be prime numbers greater than 3. Prove that both $p^2 - 1$ and $p^2 - q^2$ are divisible by 24.

Problem 5. In a city there are M soccer teams, each of which has 11 players. All the players are gathered at an airport for a trip to another country for an important game, but they are traveling "standby". There are 10 flights to their destination, and it turns out that each flight has room for exactly M players. One player will take his own helicopter to the game, rather than traveling standby on a plane. Show that at least one whole team will be sure to get to the game.