more exercises

1. Say $f : \mathbf{R}^n \to \mathbf{C}$ is measurable, and that for some $s > n/2$, $(1+|x|)^s f(x)$ is in L^2. Show that $\hat{f}$ is in C^k, for any integer $k < s - \frac{n}{2}$.

2. Say $k < -n$, that $f \in C^\infty(\mathbf{R}^n)$, and that outside some compact set, f agrees with a function which is homogneous of degree k. Show that $\hat{f}$ is smooth away from the origin. (Hint: to show that $\hat{f}$ has, say, M continuous derivatives away from 0, it is enough to show that $|\xi|^{2N}\hat{f}(\xi)$ does, for N sufficiently large.)