another problem

On $\mathbf{R}$, let D denote d/dx. Let p be a monic nth degree polynomial on $\mathbf{R}$ with real coefficients, say $p(x) = x^n + a_{n-1}x^{n-1} + \ldots + a_0$. Let $p(D)$ be the differential operator $p(D) = D^n + a_{n-1}D^{n-1} + \ldots + a_0$. Recall that there is a unique solution F on $\mathbf{R}$ of the differential equation $p(D) = 0$, with the initial conditions

$$F(0) = F'(0) = \ldots = F^{(n-2)}(0) = 0, \ F^{(n-1)}(0) = 1,$$

Suppose that $\varphi \in C_c^n(\mathbf{R})$ (that is, φ is n times continuously differentiable, and has compact support). Also let $H(x) = \chi_{[0,\infty)}$. Show that the function $\psi = \varphi * (FH)$ satisfies $p(D)\psi = \varphi$. (Here $*$ denotes convolution on $\mathbf{R}$.)
(Hint: the proof is somewhat similar to the proof that $\Delta(f * N) = f$ for suitable f.)