Leftover problems for Tuesday, October 20

1. Let H be the upper half plane $\{x + iy : y > 0\}$ in $\mathbf{C}$. Let D be an open disc centered at 0. Suppose that $f : H \cup D \to \mathbf{C}$ is holomorphic. Let L be the boundary of H, namely the real axis. Suppose also that the restriction of f to H extends to a continuous, one-to-one map, from $\overline{H}$ to $\mathbf{C}$. Suppose also that $f(L)$ is a C^1 curve. Show that $f'(0) \neq 0$.
(Hint: if not, say f has a zero of order $m \geq 2$ at 0. If $m \geq 3$, obtain a contradiction by looking at the images under f of short ray segments emanating from the origin into H. Use the "four corners lemma". Argue the case $m = 2$ separately. The hypothesis that $f(L)$ is a C^1 curve is needed only when $m = 2$.)

2. Complete the solution of problem I-6 from January 2009.
(Hint: assume it is a UFD. Obtain a contradiction from $x^3 = y^2$ (or, rather, that the coset of x cubed is the coset of y squared). If it were a UFD, argue that x would have to divide y (or, rather, that ...) Show that it cannot, by using the fact that each coset in the ring R has a unique representative of the form $p(x) + q(x)y$, where p and q are polynomials.)