

A Guideline for Solving Quadratic Inequalities.  
(Case when the quadratic polynomial can be factored.)

Here is a simple procedure you might want to follow.

1. If the coefficient of  $x^2$  is negative, multiply  $-1$  to everything to make the leading coefficient positive. When you do this, remember that the direction of the inequality changes.

Example)

$$-x^2 + 3x + 4 < 0$$

is changed to:

$$x^2 - 3x - 4 > 0$$

2. Factor the polynomial and write down the roots.  
Example)

$$x^2 - 3x - 4 > 0$$

when factored, you get

$$(x - 4)(x + 1) > 0$$

So the two roots are  $x = 4$  and  $x = -1$ .

3. If the inequality is  $<$  or  $\leq$  then  $x$  is in between the roots. If not,  $x$  is bigger than the larger root or less than the smaller root.

Example)

$$(x - 4)(x + 1) > 0$$

has solution :  $x > 4$  or  $x < -1$ .

While

$$(x - 4)(x + 1) < 0$$

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has solution :  $-1 < x < 4$ .

Also

$$(x - 4)(x + 1) \leq 0$$

has solution :  $-1 \leq x \leq 4$ .

Some more examples:

a.

$$3x^2 - 4x - 7 < 0$$

$$(3x - 7)(x + 1) < 0$$

roots:  $x = \frac{7}{3}$ ,  $-1$ .

The answer is :  $-1 < x < \frac{7}{3}$ .

b.

$$-x^2 - 6x - 5 < 0$$

$$x^2 + 6x + 5 > 0$$

$$(x + 1)(x + 5) > 0$$

roots:  $x = -5$ ,  $-1$ .

The answer is :  $x > -1$  or  $x < -5$ .

c.

$$-x^2 + 9x - 14 \geq 0$$

$$x^2 - 9x + 14 \leq 0$$

$$(x - 7)(x - 2) \leq 0$$

roots:  $x = 2$ ,  $7$

The answer is :  $2 \leq x \leq 7$ .