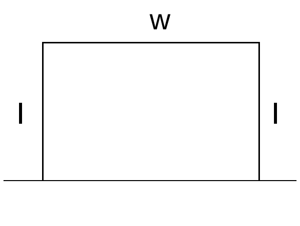
MAT123 Quiz 2

Name:

1. A farmer has 10 yard of fencing and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river.

a) Find a function that models the area of the field in one of its sides.



$10 = w + l + l = w + 2l$

Area = wl

We want Area as a function of l (or w) only.

Solving $10 = w + 2l$ for w, we get $w = 10 - 2l$.

Area $= wl = (10-2l)l = 10l - 2l^2 = -2l^2 + 10l$

Answer: Area $= -2l^2 + 10l$

b) What are the dimensions of the field when the area is 12 yd^2 ?

Area$= -2l^2 + 10l = 12$

This is a quadratic equation of l.

$-2l^2 + 10l = 12$

$0 = 2l^2 - 10l + 12$

$0 = 2(l^2 - 5l + 6)$

$0 = 2(l-2)(l-3)$

So $l = 2$ or $l = 3$.

When $l = 2$, $w = 10 - 2l = 10 - 2\times 2 = 6$ and when $l = 3$, $w = 10 - 2l = 10 - 2\times 3 = 4$.

So there are two possible dimensions,

$l = 2$ (cm) and $w = 6$ (cm)

or

$l = 3$ (cm) and $w = 4$ (cm).

c) What are the dimensions of the field when the area is maximized?

$$-\frac{b}{2a} = -\frac{10}{2\times(-2)} = \frac{10}{4} = 2.5$$

So the area is maximized when $l = 2.5$ (cm) and $w = 10 - 2l = 10 - 2\times 2.5 = 5$ (cm)

2. Let $f(x) = \dfrac{3x-4}{2x-3}$. What is $f^{-1}(x)$?

$y = \dfrac{3x-4}{2x-3}$.

Changing x and y, we get $x = \dfrac{3y-4}{2y-3}$.

So $\dfrac{x}{1} = \dfrac{3y-4}{2y-3}$, and we may cross multiply.

$x(2y-3) = 3y-4$

$2xy - 3x = 3y - 4$

We want to solve this for y, so we move to one side whatever term that contains y and move to the other side whatever term that does not contain y.

$2xy - 3y = 3x - 4$

factor left

$(2x-3)y = 3x-4$

Finally we have $y = \dfrac{3x-4}{2x-3}$.

So $f^{-1}(x) = \dfrac{3x-4}{2x-3}$. You may also have $f^{-1}(x) = \dfrac{-3x+4}{-2x+3} = \dfrac{4-3x}{3-2x}$, they are all same.

(Note: in this problem the function is inverse of itself!)