Thomas Krämer and Claude Sabbah pointed out to me that the published proof of Lemma 20.2 only works in the *regular* case. The purpose of this note is to give a correct proof for the general case. Here is the statement again.

Lemma. *Let $f\colon A \to B$ be a surjective morphism of abelian varieties, with connected fibers. If $\mathcal{N}$ is a nontrivial simple holonomic $\mathscr{D}_B$-module, then $f^*\mathcal{N}$ is a simple holonomic $\mathscr{D}_A$-module.*

Proof. Since f is smooth, $f^*\mathcal{N} = \mathscr{O}_A \otimes_{f^{-1}\mathscr{O}_B} f^{-1}\mathcal{N}$ is a holonomic $\mathscr{D}_A$-module, and so there is a surjective morphism $f^*\mathcal{N} \to \mathcal{M}$ to a nontrivial simple holonomic $\mathscr{D}_A$-module $\mathcal{M}$. We will prove the assertion by showing that it is an isomorphism.

The support $X = \operatorname{Supp}\mathcal{N}$ is an irreducible subvariety of B. As $\mathcal{N}$ is holonomic, there is a dense Zariski-open subset $U \subseteq B$ such that $X \cap U$ is nonsingular and such that the restriction $\mathcal{N}|_U$ is the direct image of a holomorphic vector bundle with integrable connection $(\mathscr{E}, \nabla)$ on $X \cap U$ [HTT08, Proposition 3.1.6]. This means that $f^*\mathcal{N}$ is supported on $f^{-1}(X)$, and that its restriction to $f^{-1}(U)$ is the direct image of $(f^*\mathscr{E}, f^*\nabla)$. We observe that, on the fibers of f over points of $X \cap U$, the latter is a trivial bundle of rank $n = \operatorname{rk}\mathscr{E}$.

Since $\mathcal{M}$ is a quotient of $f^*\mathcal{N}$, its restriction to $f^{-1}(U)$ is also the direct image of a holomorphic vector bundle with integrable connection on $f^{-1}(X \cap U)$; as a quotient of $(f^*\mathscr{E}, f^*\nabla)$, the restriction of this bundle to the fibers of f must be trivial of some rank $k \leq n$. Now let $r = \dim A - \dim B$ be the relative dimension of f. By adjunction [HTT08, Corollary 3.2.15], the surjective morphism

$$f^*\mathcal{N} \to \mathcal{M}$$

gives rise to a nontrivial morphism

$$\mathcal{N} \to \mathcal{H}^{-r} f_+\mathcal{M}$$

which must be injective because $\mathcal{N}$ is simple. Over U, the left-hand side is a vector bundle of rank n and the right-hand side a vector bundle of rank $k \leq n$; this is only possible if $k = n$. But then $f^*\mathcal{N} \to \mathcal{M}$ is an isomorphism over $f^{-1}(U)$, and since $\mathcal{M}$ is simple, we obtain a short exact sequence

$$0 \to \mathscr{K} \to f^*\mathcal{N} \to \mathcal{M} \to 0$$

where $\mathscr{K}$ is a holonomic $\mathscr{D}_A$-module whose support is contained in $f^{-1}(X \setminus X \cap U)$.

Now recall that $f^+\mathcal{N} = f^*\mathcal{N}[r]$. By adjunction [HTT08, Corollary 3.2.15], the inclusion $\mathscr{K} \to f^*\mathcal{N}$ determines a morphism $f_+\mathscr{K}[r] \to \mathcal{N}$, which factors as

$$f_+\mathscr{K}[r] \to \mathcal{H}^r f_+\mathscr{K} \to \mathcal{N}.$$

Since the support of $\mathcal{H}^r f_+\mathscr{K}$ is contained in $X \setminus X \cap U$, the morphism in question must be zero, and so $\mathscr{K} = 0$. We conclude that $f^*\mathcal{N}$ is isomorphic to $\mathcal{M}$. $\square$

References

[HTT08] R. Hotta, K. Takeuchi, and T. Tanisaki, *D-modules, perverse sheaves, and representation theory*, Progress in Mathematics, vol. 236, Birkhäuser Boston Inc., Boston, MA, 2008.