1. a. true

b. true

c. false

d. false, $\dim(\text{im}(A)) = 3$.

e. true

2. The dimension is 2. They are linearly independent.

3. $\det(A) = -2(k-3)$. A is not invertible for $k = 3$.

4. a. $T(\begin{bmatrix}0\\2\end{bmatrix}) = (-3)\begin{bmatrix}0\\2\end{bmatrix} + (2)\begin{bmatrix}1\\1\end{bmatrix}$ and $T(\begin{bmatrix}1\\1\end{bmatrix}) = (-1)\begin{bmatrix}0\\2\end{bmatrix} + (3)\begin{bmatrix}1\\1\end{bmatrix}$. So with respect to the basis $\mathcal{B} = \{\begin{bmatrix}0\\2\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$, T has matrix $B = \begin{bmatrix}-3 & -1\\2 & 3\end{bmatrix}$.

The change of basis from $\mathcal{B}$ to the standard is $S = \begin{bmatrix}0 & 1\\2 & 1\end{bmatrix}$. Recall,

$$B = S^{-1}AS$$

where A is the matrix of T with respect to the standard basis. Thus

$$A = SBS^{-1} = \begin{bmatrix}0 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}-3 & -1\\2 & 3\end{bmatrix}\begin{bmatrix}-\frac{1}{2} & \frac{1}{2}\\1 & 0\end{bmatrix} = \begin{bmatrix}2 & 1\\3 & -2\end{bmatrix}.$$

5. $\ker(C) = \{\begin{bmatrix}-3a\\b\\a\\0\\a\end{bmatrix} : a, b \in \mathbb{R}\}$, and

$\dim(\text{im}(C)) = 3$.

6. $a = b = 0$ and $c = 1$.

7. Eigenvectors $\lambda = 2, 2i, -2i$, since the characteristic polynomial is $f_B(\lambda) = (\lambda^2+4)(2-\lambda)$.

The eigenspace of $\lambda = 2$ is

$$E_2 = \ker(\begin{bmatrix}-2 & 0 & 4\\0 & 0 & 0\\-1 & 0 & -2\end{bmatrix}) = \text{span}(\begin{bmatrix}0\\1\\0\end{bmatrix}).$$

8. Eigenbasis $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}\right\}$

9.a. $\det(A) = 2$, $\det(B) = 4$, $\det(C) = -5$ so they are invertible.

b. $ABB^{-1}BCC^{-1}A^{-1} = ABA^{-1} = \begin{bmatrix} 4 & -12 \\ 1 & -2 \end{bmatrix}$, by direct computation.

10. a. $\vec{v}_3 = 5 \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} -2 \\ -4 \\ 2 \\ 1 \end{bmatrix}$

b. $\vec{u}_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$, and $\vec{u}_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$

11. a. $A = \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix}$.

b. Eigenvalues are $\lambda = 0, 1$.

$\lambda = 1$ has the eigenvector $\vec{u} = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$,

$\lambda = 0$ has the eigenvector $(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$