§1.2

6. The system is already in row reduced form with leading variables x_1, x_3, x_4 and free variables x_2, x_5. Let $s = x_2$ and $t = x_5$. The set of solutions is

$$\left\{ \begin{bmatrix} 3+7s-t \\ s \\ 2+2t \\ 1-t \\ t \end{bmatrix} : s,t \in \mathbb{R} \right\}.$$

7. The reduced row echelon form of the system is

$$\left| \begin{array}{ccccccc} x_1 & +2x_2 & & & & = 0 \\ & & x_3 & & & = 0 \\ & & & x_4 & & = 0 \\ & & & & x_5 & = 0 \end{array} \right|.$$

We have leading variables x_1, x_3, x_4, x_5 and free variable x_2. If $t = x_2$ then every solution is

$$\begin{bmatrix} -2t \\ t \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ with } t \in \mathbb{R}.$$

8. The rref of the system is

$$\left| \begin{array}{cccc} x_2 & & -x_5 & = 0 \\ & x_4 & +2x_5 & = 0 \end{array} \right|.$$

The leading variables are x_2, x_4 and the free variables are x_1, x_3, x_5. We have

$$x_2 = x_5$$
$$x_4 = 2x_5$$

Set $t = x_5$, then $\begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} t \\ 2t \\ t \end{bmatrix}$ for $t \in \mathbb{R}$ where of course x_1, x_3 can be equal to anything.

27. No. rref $= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ which is different than the row reduced echelon matrix shown.

28. Notice that the operation of subtracting a row from another is reversible, so one doesn't loose any information in the system of equations.

34. We have $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = 0$ which is equivalent to

$$x_1 + 3x_2 - x_3 = 0$$

which is already in reduced form. Let $s = x_2, t = x_3$, then the space of solutions is

$$\left\{ \begin{bmatrix} -3s + t \\ s \\ t \end{bmatrix} : s, t \in \mathbb{R} \right\}.$$

35. This is equivalent to the system

$$\left| \begin{array}{llll} x_1 & +x_2 & +x_3 & +x_4 = 0 \\ x_1 & +2x_2 & +3x_3 & +4x_4 = 0 \\ x_1 & +9x_2 & +9x_3 & +7x_4 = 0 \end{array} \right|$$

with augmented matrix form

$$\begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 1 & 2 & 3 & 4 & | & 0 \\ 1 & 9 & 9 & 7 & | & 0 \end{bmatrix}.$$

This has rref

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & | & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & | & 0 \\ 0 & 0 & 1 & \frac{9}{4} & | & 0 \end{bmatrix},$$

so we have leading variables x_1, x_2, x_3, and x_4 is a free variable. Let $t = x_4$. The set of solutions is

$$\left\{ \begin{bmatrix} -\frac{1}{4}t \\ \frac{3}{2}t \\ -\frac{9}{4}t \\ t \end{bmatrix} : t \in \mathbb{R} \right\}.$$

§1.3

26. Since $A\vec{x} = \vec{b}$ has a unique solution, it is consistent and the system has no free variables. Thus $\text{rank}(A) = 3$ and $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. The system $A\vec{x} = \vec{c}$ has either precisely one solution or none, depending on whether it is consistent.

30. Note that $\text{rank}(A) = 1$ if, and only if, the rows of A are proportional to each other.

$$\begin{bmatrix} \frac{2}{5} & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 \end{bmatrix}$$

does the job.

34. a. $A\vec{e_1} = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$, $A\vec{e_2} = \begin{bmatrix} b \\ e \\ h \end{bmatrix}$, $A\vec{e_3} = \begin{bmatrix} c \\ f \\ k \end{bmatrix}$.

b. $B\vec{e_1} = \vec{v_1}$, $B\vec{e_2} = \vec{v_2}$, $B\vec{e_3} = \vec{v_3}$.

36. $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

48. a. $A(\vec{x_1} + \vec{x_h}) = A\vec{x_1} + A\vec{x_h} = \vec{b} + \vec{0} = \vec{b}$.

b. $A(\vec{x_2} - \vec{x_1}) = A\vec{x_2} - A\vec{x_1} = \vec{b} - \vec{b} = \vec{0}$.

c. The set of solutions to $A\vec{x} = \vec{b}$ is $\{\vec{x_1} + \vec{x} : A\vec{x} = 0\}$.

p.38(true/false)

1. true

2. false

3. false

4. true

5. true

6. false

7. false

8. false

9. true

10. true

11. false

12. false

13. true

14. true

15. true

16. true

17. true

18. true

19. false

20. false

§2.1 **44.** Yes, it is linear. If $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$, then the matrix of T is

$$\begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}.$$

§2.3 **8.** Invertible with matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

10. Invertible with inverse $\begin{bmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$.

42. A permutation matrix is invertible. Its inverse is also a permutation matrix. If A is a permutation matrix then it can be show that A^{-1} is the transpose, see p.213 for definition, of A.

§2.4 **10.** $\begin{bmatrix} 0 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

14. BC, BD, CD, DB, DE, EB are the matrix products which are defined.

20. $(A-B)(A+B) = A^2 - BA + AB - B^2 = A^2 - B^2$ only if $AB = BA$, i.e. the matrices commute, which is not necessarily true.

22. $ABA^{-1} = B$ if and only if $AB = BA$ which is usually not the case.

24.

$$\begin{aligned} (I_n + A)(I_n + A^{-1}) &= I_n^2 + AI_n + I_nA^{-1} + AA^{-1} \\ &= I_n + A + A^{-1} + I_n \\ &= 2I_n + A + A^{-1} \end{aligned}$$

So it is true.

p. 97

1. true, is has matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

2. false, $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$, which is a product of scaling by $\frac{1}{\sqrt{2}}$ and rotation counterclockwise by $\frac{\pi}{4}$.

3. true

4. true

5. false

6. true, see p.85 fact 2.4.9

7. false, AB is 3×5.

8. false

9. true

10. true $AA^{-1} = A^{-1}A = I_n$.

11. false

12. true

13. true. Note that for any $a \in \mathbb{R}$, $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}^2 = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$.

14. false

15. true. If det $= k(k-6) - 5(-2) = k^2 - 6k + 10 = 0$, then the quadratic formula gives

$$k = \frac{6 \pm \sqrt{-4}}{2},$$

which is not real.