MAT 211 (Intro. Linear Algebra), Sec. 2, Spring 2000
Final Exam. May 10, 2000

VERSION A

Name: ______________________________ ID # ________________

Problem	Max points	Grade
1	35	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
8	15	
9	20	
10	20	
11	20	
Total	200	

Instructions: Write legibly and use meaningful mathematical formalism. Show all of your work, in order to receive full or partial credit. If you need more space, you can always use the back of the pages. In this case, make a clear reference to the continuation of your work.

1. **[5 $\times$ 7 pts]** True or false. Give a short explanation for your answers. **No credit** will be given without an explanation.

   (a) Linear algebra is the coolest subject you've ever studied.

   (b) There are no more than 4 linearly independent vectors in $\mathbb{R}^4$.

   (c) If a square matrix A has all zeros in the diagonal, then $\det(A) = 0$.

   (d) Let A be a 3×4 matrix. If $dim(ker(A)) = 1$, then $dim(Im(A)) = 2$.

   (e) The identity matrix I_n admits an eigenbasis.

2. **[15 pts]** What is the dimension of

$$span \left\{ \begin{bmatrix} -3 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\} ?$$

Explain.

3. **[15 pts]** For what values of k is the following matrix **not** invertible?

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & -1 & -2 & k \end{bmatrix}.$$

4. **[15 pts]** A linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is given such that

$$T\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -4 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

(a) Compute the matrix corresponding to T

(b) Draw a sketch of $T(\vec{e_1})$ and $T(\vec{e_2})$ in $\mathbb{R}^2$.

5. **[15 pts]** Compute $ker(C)$ for

$$C = \begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 1 & 0 & 2 & 2 & 1 \\ 3 & 0 & 0 & 5 & 9 \\ 2 & 0 & 0 & 5 & 6 \end{bmatrix}.$$

Also, what is $dim(Im(C))$?

6. **[15 pts]** Fix a, b, c in such a way that the following matrix is orthogonal:

$$U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & a \\ 1/\sqrt{2} & -1/\sqrt{2} & b \\ 0 & 0 & c \end{bmatrix}.$$

7. **[15 pts]** Find all the (real and complex) eigenvalues and all the (real) eigenvectors of the matrix

$$B = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix}.$$

8. **[15 pts]** Calculate an eigenbasis (if any) of the matrix

$$C = \begin{bmatrix} 3 & 5 & -5 \\ 0 & 3 & 0 \\ 0 & 5 & -2 \end{bmatrix}$$

9. **[20 pts]** Define the three matrices

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -6 \\ 2 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}.$$

(a) Check that they are invertible.

(b) Calculate $ABB^{-1}BCC^{-1}A^{-1}$.

10. **[20 pts]** Consider the vectors

$$\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} -2 \\ -4 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} -1 \\ -2 \\ 6 \\ 3 \end{bmatrix}.$$

(a) Show that $\vec{v_3}$ is a linear combination of $\vec{v_1}$ and $\vec{v_2}$. [*This means that you must write out explicitly this linear combination.*]

(b) Having eliminated $\vec{v_3}$, apply Gram-Schmidt to find an orthonormal basis of $span\{\vec{v_1}, \vec{v_2}\}$.

11. **[20 pts]** In $\mathbb{R}^2$ consider L, the line spanned by the vector $\vec{u} = (2/\sqrt{5}, 1/\sqrt{5})$.

(a) Write the matrix A corresponding to $proj_L$.

(b) Find the eigenvalues and eigenvectors of A.

(c) In a graph, plot L and the eigenvectors.