Final Exam
MAT 211

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Last Name, First Name

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I.D.#

Exam rules: No books, notes, calculators, pagers or cell-phones.

There are ten questions, each of which is worth 10 points. Show your work, and write your final answers in the indicated spaces.

1. Let θ denote the **angle** between the vectors $\vec{v} = (1, 2, 3)$ and $\vec{w} = (3, 2, 1)$ in $\mathbb{R}^3$. Compute $\cos\theta$.

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Answer

2. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation with standard matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

Compute $T(2, 2, 0, -1)$.

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Answer

3. Let $S : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation with standard matrix

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation with standard matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Compute the standard matrix of the *composition* $T \circ S : \mathbb{R}^3 \to \mathbb{R}^3$.

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Answer

4. The linear subspace of $\mathbb{R}^3$ spanned by the vectors $\vec{v} = (1, 2, 3)$ and $\vec{w} = (3, 2, 1)$ is a plane P. Find the equation for P.

Answer

5. Compute the **determinant** of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 1 & 4 & 5 \\ 3 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

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Answer

6. Consider the vectors

$$\begin{aligned} \vec{v}_1 &= (1,2,3,4) \\ \vec{v}_2 &= (5,6,7,8) \\ \vec{v}_3 &= (4,3,2,1) \\ \vec{v}_4 &= (1,1,1,1) \end{aligned}$$

in $\mathbb{R}^4$. Compute the **dimension** of $\text{span}\{v_1, v_2, v_3, v_4\}$.

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Answer

7. Are the given vectors linearly independent, or linearly dependent? Indicate your answers by writing **I** (for *independent*) or **D** (for *dependent*) in the answer blanks. Support each answer with a calculation.

______(a) $(2, 2, 3), (1, 1, 7), (3, 3, 2)$ in $\mathbb{R}^3$.

______(b) $1 + x,\ 2 + x^2,\ 3 + x^3$ in $\mathcal{P}_3$.

______(c) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ in M_{22}

______(d) $(1, 2, 3), (1, 2, 7), (3, 3, 2)$ in $\mathbb{R}^3$.

______(e) $1 + x^3,\ 2 + x^3,\ 3 + x^3$ in $\mathcal{P}_3$.

8. Compute the area of the parallelogram in $\mathbb{R}^3$ whose corners are $(1, 1, 1)$, $(1, 3, 5)$, $(2, 4, 6)$, and $(2, 6, 10)$.

Answer

9. Find the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}$$

Answer

10. Do the vectors

$$\begin{aligned}
\vec{v}_1 &= (1,2,3,4)\\
\vec{v}_2 &= (0,1,1,1)\\
\vec{v}_3 &= (4,3,2,1)\\
\vec{v}_4 &= (1,1,1,0)
\end{aligned}$$

form a basis for $\mathbb{R}^4$? Justify your answer with a calculation.