

Renormalization
and
Non-Rigidity

A Dissertation Presented

by

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to

The Graduate School

in Partial Fulfillment of the

Requirements

for the Degree of

Doctor of Philosophy

in

Mathematics

Stony Brook University

December 2008

Abstract of the Dissertation

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The aim of the thesis is to study the renormalization of unimodal maps with low smoothness and the dynamics of Hénon renormalization.

M. Feigenbaum and by P. Coullet and C. Tresser in the nineteen-seventieth to study the asymptotic small scale geometry of the attractor of one-dimensional systems which are at the transition from simple to chaotic dynamics. This geometry turns out to not depend on the choice of the map under rather mild smoothness conditions. The existence of a unique renormalization fixed point which is also hyperbolic among generic smooth enough maps plays a crucial role in the corresponding renormalization theory. The uniqueness and hyperbolicity of the renormalization fixed point were first shown in the holomorphic context, by means that generalize to other renormalization operators. It was then proved that in the space of $C^{2+\alpha}$ unimodal maps, for $\alpha > 0$, the period doubling renormalization fixed point is hyperbolic as well.

In this thesis work we study what happens when one approaches from below the minimal smoothness thresholds for the uniqueness and for the hyperbolicity of the period doubling renormalization generic fixed point. Indeed, our main result states that in the space of C^2 unimodal maps the analytic fixed point is not hyperbolic and that the same remains true when adding enough smoothness to get a priori bounds. In this smoother class, called $C^{2+|\cdot|}$, the failure of hyperbolicity is tamer than in C^2 . Things get much worse with just a bit less of smoothness than C^2 as then even the uniqueness is lost and other asymptotic behavior become possible. Furthermore, we show that the period doubling renormalization operator acting on the space of C^{1+Lip} unimodal maps has infinite topological entropy.

The second part of the thesis work is devoted to the renormalization of Hénon maps. It was shown that for strongly dissipative Hénon maps, there is a short curve in the parameter space which consists of infinitely renormalizable Hénon maps of period doubling type. In this thesis we study numerically, the extension of this curve in the parameter space up to the conservative map. More precisely, we describe the combinatorial changes which occur along this curve. The second part of this study is to describe, how the one-dimensional Cantor set deforms into the Cantor set of conservative map. To show this we compute the distribution of angles of the line fields along the Cantor set and explain how this geometry becomes more complicated for maps close to the infinitely renormalizable conservative maps.