Stony Brook University

MAT 132 - PRACTICE FINAL

Summer 2010

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

INSTRUCTOR: Caner Koca

DURATION: **3 HOURS** (9:30am-12:30am)

IMPORTANT INSTRUCTIONS:

- There are 7 questions in this exam. You are allowed to attempt to solve the following 4 problems:
  – Problem 1
  – One problem you choose from 2, 3
  – One problem you choose from 4, 5
  – One problem you choose from 6, 7

- In the table below, please cross out the other 3 problems you don't want to be graded.

QUESTION	VALUE	SCORE
1	40	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
TOTAL	100	

1. (40 points) Determine whether the following series are convergent or divergent. In each case, provide a complete rigorous argument.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(4n)}{4^n}$ $\boxed{\text{convergent}}$

(b) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$ $\boxed{\text{convergent}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ $\boxed{\text{divergent}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n^2}$ $\boxed{\text{divergent}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^n}{3^{n+1}}$ $\boxed{\text{divergent}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \arctan n$ $\boxed{\text{divergent}}$

(g) $\displaystyle\sum_{n=5}^{\infty} \frac{1}{\sqrt{n-4}}$ $\boxed{\text{divergent}}$

(h) $\displaystyle\sum_{n=1}^{\infty} \sin(\frac{1}{n})$ $\boxed{\text{divergent}}$

2. (20 points) Find the solution of the differential equation satisfying given initial values.

(a) $y'' - 6y' + 9y = 0, \quad y(0) = 3, \quad y'(0) = 1$

$$\boxed{y = 3e^{3x} - 8xe^{3x}}$$

(b) $y'' - 4y' + 5 = 0, \quad y(\frac{\pi}{2}) = 1, \quad y'(\frac{\pi}{2}) = 2$

$$\boxed{y = 2(e^{-\pi} - 1)e^{2x}\cos x + e^{-\pi}e^{2x}\sin x}$$

3. (20 points) (a) [10pts] Find the orthogonal trajectories of the family of curves given by $y^2 = kx^3$.

$\boxed{3y^2 - 2x^2 = k}$

(b) [10pts] A tank contains 1000L of pure water. Brine that contains 0.05kg of salt per liter of water enters the tank at a rate of 5 L/min. Brine that contains 0.04kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank after t minutes?

$\boxed{y(t) = \frac{130}{3}(1 - e^{-3t/200})}$

4. (20 points) Compute the following limits:

(a) $\lim_{n\to\infty} \dfrac{e^{n+2}}{\pi^{n-1}} =$ $\boxed{0}$

(b) $\lim_{n\to\infty} \left(\dfrac{n+1}{n}\right)^{2n} =$ $\boxed{e^2}$

(c) $\lim_{n\to\infty} a_n$, where $\quad a_n = 3 - \dfrac{1}{a_{n-1}} \quad$ for $n \geq 2$ and $\quad a_1 = 1$

(Hint: Show that the sequence is increasing, and $a_n < 3$ for all n.) $\boxed{\dfrac{3-\sqrt{5}}{2}}$

5. (20 points) (a) Is the series $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ convergent or divergent? Why? If it is convergent, evaluate the sum (that is, find the number which the series is equal to). $\boxed{\text{convergent, } \frac{\sqrt{2}}{\sqrt{2}-1}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = ?$ $\boxed{\frac{11}{6}}$

(c) Express the number $0.\overline{25}$ as a ratio of integers. $\boxed{\frac{25}{99}}$

6. (20 points) Find the interval of convergence and radius of convergence of the following series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2+1}$ $\boxed{[1,3],\ R=1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{2^n} x^n$ $\boxed{\{0\},\ R=0}$

7. (20 points) (a) [10pts] Find a power series representation of the function $f(x) = \dfrac{x}{2x^2+1}$. Determine its interval of convergence.

$$\boxed{\sum_{n=0}^{\infty}(-1)^n 2^n x^{2n}, \quad \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \quad R = \frac{1}{\sqrt{2}}}$$

(b) [10pts] Find the MacLaurin series expansion of the function $f(x) = x^2 \ln(1+x^3)$. Determine its interval of convergence.

$$\boxed{\sum_{n=0}^{\infty}(-1)^n \frac{x^{3n+5}}{n+1}, \quad (-1, 1], \quad R = 1}$$