

Stony Brook University
MAT 132 - PRACTICE FINAL
Summer 2010

NAME (please print legibly): _____

Your University ID Number: _____

INSTRUCTOR: Caner Koca

DURATION: **3 HOURS** (9:30am-12:30am)

IMPORTANT INSTRUCTIONS:

- There are 7 questions in this exam. You are allowed to attempt to solve the following 4 problems:
 - Problem 1
 - One problem you choose from 2, 3
 - One problem you choose from 4, 5
 - One problem you choose from 6, 7

- In the table below, please cross out the other 3 problems you don't want to be graded.

QUESTION	VALUE	SCORE
1	40	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
TOTAL	100	

1. (40 points) Determine whether the following series are convergent or divergent. In each case, provide a complete rigorous argument.

$$(a) \sum_{n=1}^{\infty} \frac{\sin(4n)}{4^n}$$

$$(b) \sum_{n=1}^{\infty} n \left(\frac{2}{3}\right)^n$$

$$(c) \sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$(d) \sum_{n=1}^{\infty} \frac{(-3)^n}{n^2}$$

$$(e) \sum_{n=1}^{\infty} \frac{\pi^n}{3^{n+1}}$$

$$(f) \sum_{n=1}^{\infty} \arctan n$$

$$(g) \sum_{n=5}^{\infty} \frac{1}{\sqrt{n-4}}$$

$$(h) \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

2. (20 points) Find the solution of the differential equation satisfying given initial values.

(a) $y'' - 6y' + 9y = 0$, $y(0) = 3$, $y'(0) = 1$

(b) $y'' - 4y' + 5 = 0$, $y(\frac{\pi}{2}) = 1$, $y'(\frac{\pi}{2}) = 2$

3. (20 points) (a) [10pts] Find the orthogonal trajectories of the family of curves given by $y^2 = kx^3$.

(b) [10pts] A tank contains 1000L of pure water. Brine that contains 0.05kg of salt per liter of water enters the tank at a rate of 5 L/min. Brine that contains 0.04kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank after t minutes?

4. (20 points) Compute the following limits:

(a) $\lim_{n \rightarrow \infty} \frac{e^{n+2}}{\pi^{n-1}} =$

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^{2n} =$

(c) $\lim_{n \rightarrow \infty} a_n$, where $a_n = 3 - \frac{1}{a_{n-1}}$ for $n \geq 2$ and $a_1 = 1$
(Hint: Show that the sequence is increasing, and $a_n < 3$ for all n .)

5. (20 points) (a) Is the series $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ convergent or divergent? Why? If it is convergent, evaluate the sum (that is, find the number which the series is equal to).

(b) $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = ?$

(c) Express the number $0.\overline{25}$ as a ratio of integers.

6. (20 points) Find the interval of convergence and radius of convergence of the following series:

$$(a) \sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2+1}$$

$$(b) \sum_{n=1}^{\infty} \frac{(2n)!}{2^n} x^n$$

7. (20 points) (a) [10pts] Find a power series representation of the function $f(x) = \frac{x}{2x^2 + 1}$. Determine its interval of convergence.

(b) [10pts] Find the MacLaurin series expansion of the function $f(x) = x^2 \ln(1 + x^3)$. Determine its interval of convergence.