MATH 534 HOMEWORK ASSIGNMENT 2

1. Let M, N be normal subgroups of a group G, if $M \cap N = 1$, then $ab = ba$ for any $a \in M$, $b \in N$.

2. If $f : G \to H$ is a group homomorphism, g is an element of finite order in G, then $ord(f(g))|ord(g)$,

3. Let $N \triangleleft G$, $g \in G$, if $ord(g)$ and $|G/N|$ are coprime, then $g \in N$

4. Let G be a group and H a normal subgroup, then G is solvable if and only if H and G/H are solvable.

5. A group G is solvable iff $G^{(n)} = 1$ for some n.

6. A nilpotent group is solvable.

7. Find all subgroups of $\mathbb{Z}$.

8. If a finite group has a unique maximal subgroup, then it is a cyclic group of order p^r for some prime p.

9. Let G be a group such that $Aut(G)$ is cyclic, prove that G is abelian.

10. Classifying groups of order 4 upto isomorphism.

11. Any group of order p^2 is abelian, where p is a prime.

12. Let H be a proper sungroup of a finite group G, then $G \neq \cup_{g \in G} gHg^{-1}$. Is this true for infinite groups?

13. If α is an automorphism of a finite group G, let $I = \{g \in G | \alpha(g) = g^{-1}\}$. If $|I| > \frac{3}{4}|G|$, then G is abelian, if $|I| = \frac{3}{4}|G|$, then G has a abelian normal subgroup of index 2.

14. Prove that two elements of the symmetric group S_n are conjugate iff they have the same type.

15. Find $Z(S_n)$.

16. Find all normal subgroups of S_4.

17. Let G be a group of order p^r, where p is a prime, prove that the number of non-normal subgroups is a multiply of p.

18. Let G be a group of order $2^n m$, where m is an odd number. If G has an element of order 2^n, then G has a normal subgroup of index 2^n.

19. If p is the minimal prime divisor of $|G|$, A is a normal subgroup of order p, then $A \subset Z(G)$.