

# Chapter 5 Review/Extra Credit Problems

Remember, no credit will be given for answers without justification.

Instructions: Do #1, either #2 or #3, and either #4 or #5.

1) Let

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

a) (5 pts) Compute  $e^{\mathbf{A}}$ .

b) (5 pts) Compute  $e^{t\mathbf{A}}$ .

c) (5 pts) Solve the initial value problem

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}' = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$$

In the following two problems, you will consider a system  $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$ . Answer the following questions:

a) (10 pts) Find the complimentary solution

b) (5 pts) What is your 'guess' for the particular solution,  $\mathbf{x}_p$ ?

Special instructions: Be *extremely* specific. Do not write, for example,  $\mathbf{x}_p = t\mathbf{a} + \mathbf{b}$ , but instead write  $\mathbf{x}_p = t \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ , or maybe  $\mathbf{x}_p = \begin{pmatrix} ta_1 + b_1 \\ ta_2 + b_2 \end{pmatrix}$ .

c) (10 pts) Plug in your 'guess' for  $\mathbf{x}_p$ . You will get a system of 4 equation and 4 unknowns. Write this system in augmented matrix form.

d) (5 pts) Find  $\mathbf{x}_p$ .

$$2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 2 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$$

$$3) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} -1 & 2 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} t \\ 1 + t \end{pmatrix}$$

In the following two problems, you will consider a system  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ . Answer the following questions:

a) (10 pts) Find all the generalized eigenvectors of  $A$ .

b) (5 pts) Find the general solution.

c) (10 pts) Find  $\Phi(t)$  and compute  $\Phi(0)^{-1}$ .

d) (5 pts) Find the solution, given  $\mathbf{x}(0) = (-1 \ 1 \ 0 \ 1)^T$ .

4)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}' = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

5)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}' = \begin{pmatrix} -1 & -1 & -1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$