

# Extra Credit/Review Problems II

## Instructions

Your best 4 problems will be counted.

A full 30 points counts as one homework assignment.

No credit will be given for answers without justification.

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Evaluate (3 points each).

1)  $\int_1^{\sqrt{3}} x^{-4} \sqrt{x^2 + 1} dx$

*Ans:*  $\frac{8}{3} \left( \frac{1}{2\sqrt{2}} - \frac{1}{3\sqrt{3}} \right)$

2)  $\int_0^1 x^2 (1 + x^2)^{-\frac{5}{2}} dx$

*Ans:*  $\frac{\sqrt{2}}{12}$

3)  $\int_0^1 (1 + x^2)^{-\frac{3}{2}} dx$

4)  $\int_1^{\sqrt{3}} x^4 (1 + x^2)^{-\frac{7}{2}} dx$

Evaluate (3 points each)

5)  $\int_e^{e^e} \frac{1}{x \ln(x)} dx$

Ans: 1

6)  $\int_e^{e^{e^e}} \frac{1}{x \ln(x) \ln(\ln(x))} dx$

Ans: 1

7)  $\int_e^{e^{e^2}} \frac{1}{x \ln(x) (\ln(\ln(x)))^2} dx$

8)  $\int_e^{e^{e^3}} \frac{(\ln(\ln(x)))^2}{x \ln(x)} dx$

Let  $\Gamma(x)$  be the function of  $x$  defined by

$$\Gamma(x) = \int_0^1 s^x e^{-s} ds.$$

9) (3 pts) Evaluate  $\Gamma(1)$

Ans: 1

10) (3 pts) Evaluate  $\Gamma(2)$

Ans: 2

11) (5 pts) Evaluate  $\Gamma(3)$

12) (5 pts) Evaluate  $\Gamma(4)$

13) (10 pts) Find the Taylor series centered at  $x = -1$  for  $f(x) = (2 - x)^{-2}$ , and determine the radius of convergence.

14) (10 pts) Find the Taylor series centered at  $x = 1$  for  $f(x) = x^2 \ln(x)$ , and determine the radius of convergence.