

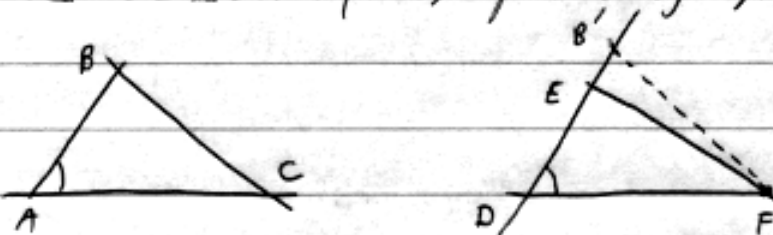
axiom B-3: satisfied because B-3 is already satisfied in the euclidean plane.

**#3**: This statement is clearly a consequence of the Angle-Side-Angle criterion.

So let's prove the A-S-A criterion (saying that  $\triangle ABC \cong \triangle DEF$  if  $\angle A \cong \angle D$ ,  $\angle C \cong \angle F$  and  $AC \cong DF$ ):

Proof of A-S-A criterion:

- 1) By axiom C-1, there is a unique pt  $B'$  on ray  $\vec{DE}$  s.t. that  $DB' \cong AB$
- 2) Thus  $\triangle ABC \cong \triangle DB'F$  (S-A-S criterion).
- 3) Then  $\angle DFB' \cong \angle C$  (consequence of  $\triangle ABC \cong \triangle DB'F$ ).
- 4) From C-4, we know that ray  $\vec{FB'} = \text{ray } \vec{FE}$  (because the angles at F are both equal to  $\angle C$ ).
- 5) Therefore  $B' = E$  (unique pt of intersection of  $\vec{DE}$  and  $\vec{FE} = \vec{FB'}$ ).
- 6) Conclusion:  $\triangle ABC \cong \triangle DEF$ . (in 2) replace  $B'$  by E).



**#4**: We know  $\angle ABC \cong \angle DEF$ , we want to prove  $\angle CBG \cong \angle FEH$ .

- 1) We can choose D, F, H so that  $AB \cong DE$ ,  $CB \cong FE$ ,  $BG \cong EH$  (by axiom C-1).
- 2) Thus  $\triangle ABC \cong \triangle DEF$  (S-A-S axiom).
- 3) Thus  $AC \cong DF$  and  $\angle A \cong \angle D$
- 4) Then  $AG \cong DH$  (axiom C-3).
- 5) So  $\triangle ACG \cong \triangle DFH$  (S-A-S axiom with angle  $\angle A$  and  $\angle D$ )
- 6) Thus  $CG \cong FH$ ,  $\angle G \cong \angle H$
- 7) Then  $\triangle CBG \cong \triangle FEH$  (S-A-S again)
- 8) Therefore  $\angle CBG \cong \angle FEH$ .

