Practice Midterm I

Name:

Student I.D:

Problem 1. (25 points) Assume that the Incidence axioms and the Betweenness axioms are satisfied and prove the following statement:

Given $A * B * C$ and $A * C * D$, then we must have $B * C * D$ and $A * B * D$.

(Justify each step by saying which axiom you use, for example, "by I3, we know that...")

Problem 2. (25 points) Consider the following geometry:

Points: they are couples of the form $(\frac{a}{2^n}, \frac{b}{2^m})$ where a, b, n, m are integers;

Lines: they are just the standard euclidean lines joigning two such points in $\mathbb{R}^2$.

Show that the incidence axioms are satisfied and also the first three betweenness axioms.

Problem 3. (25 points) Assume that the Incidence axioms, the Betweenness axioms and the Congruence axioms are satisfied. Prove the following statement:

If in the triangle $\triangle$ABC *we have* angle $\lhd B \cong$ angle $\lhd C$ *then* AB $\cong$ AC *and the triangle* $\triangle$ABC *is isosceles.*

Problem 4. (25 points) Assume that the Incidence axioms, the Betweenness axioms and the Congruence axioms are satisfied. Prove the following statement:

Supplements of congruent angles are congruent.

(You are allowed to use these axioms, and also the following congruence propositions in the list of axioms: P3.10, P3.11, P3.12. You might want to use the Side-Angle-Side axiom, so using your angles you should build similar triangles on them...)