

## PRACTICE MIDTERM II

Name:

Student I.D:

**Problem 1. (25 points)**

1. Prove that in any Hilbert space every angle has a unique bisector. (You can use the existence and unicity of the midpoint of a segment).
2. Consider a triangle  $\triangle ABC$ . Using the above result, bisect the angle  $\sphericalangle A$ . Explain why the bisector meets segment  $BC$  at some point  $D$ . Use this point  $D$  to reprove the triangle inequality (namely  $\overline{AB} + \overline{AC} > \overline{BC}$ ).

**Problem 2. (25 points)** Prove that Hilbert's Euclidean parallel postulate is equivalent to the following proposition:

if  $t$  is a transversal to  $l$  and  $m$ ,  $l$  parallel to  $m$ , and  $t \perp l$  then  $t \perp m$ .

**Problem 3. (25 points)** Prove that in any Hilbert plane there exists one triangle that is not isosceles.

**Problem 4. (25 points)** In any Hilbert plane, prove that the interior of a circle is a convex set.