The actual midterm II will be shorter than the first one (it doesn't mean easier...)! There might be 3 problems.

Problem 1. *Let* $\epsilon = 1 + \sqrt{2}$. *Write* $\epsilon^n = u_n + v_n\sqrt{2}$. *Show that* $u_n^2 - 2v_n^2 = \pm 1$.

Problem 2. Structure of the invertible elements in $\mathbb{Z}[\sqrt{2}]$.

1. *Show that there is no invertible element* $\alpha \in \mathbb{Z}[\sqrt{2}]$ *such that* $1 < \alpha < 1 + \sqrt{2}$.
2. *Deduce that any invertible element (greater than 0) of* $\mathbb{Z}[\sqrt{2}]$ *is a power of* $1 + \sqrt{2}$.

Problem 3. *Let* R *be the ring* $\mathbb{Q}[\alpha]$ *(meaning all the* $P(\alpha)$, *where* P *is a polynomial with coefficients in* $\mathbb{Q}$*), where* α *is a number satisfying* $\alpha^3 - \alpha^2 + \alpha + 2 = 0$.

1. *express* $(\alpha^2 + \alpha + 1).(\alpha^2 - \alpha)$ *in the form* $a\alpha^2 + b\alpha + c$, *where* a, b, c *are in* $\mathbb{Q}$.
2. *express* $(\alpha - 1)^{-1}$ *in the form* $a\alpha^2 + b\alpha + c$, *where* a, b, c *are in* $\mathbb{Q}$.

Problem 4. *Let* $f : A \to B$ *be a ring morphism. Show that for any prime ideal* $\mathcal{P}$ *in* B, *then* $f^{-1}(\mathcal{P})$ *is a prime ideal of* A.