MAT 360 05/06/2008

Practice Final

Name:

Student I.D:

Problem 1. (15 points)

In a Hilbert plane, prove that if D is an exterior point of ΔABC, then there exists a line (DE) through D that is contained in the exterior of ΔABC.

Problem 2. (15 points) In a Hilbert plane, prove that a line cannot be contained in the interior of a triangle.

Problem 3. (10 points) Assume that you have a geometry with only the incidence and the betweenness axioms: prove that every line has at least five points on it.

Problem 4. (15 points)

Let $\mathcal{M}$ be a projective plane (see previous exams for the definition). Define a new geometry $\mathcal{M}'$ by taking as "points" of $\mathcal{M}'$ the lines of $\mathcal{M}$ and as "lines" of $\mathcal{M}'$ the points of $\mathcal{M}$, with the same incidence properties. Prove that $\mathcal{M}'$ is also a projective plane.

Problem 5. (15 points)

Construct a model of incidence geometry with a finite number of points, that has neither the elliptic, hyperbolic nor euclidean parallel properties. (Basically you want to have some points having only one parallel through them to a given line, some other points with several parallels through them, etc...)

Problem 6. (15 points)

In the Poincare model of the hyperbolic plane, prove that inversions truly "are" hyperbolic reflections.

Let's recall the definition of the hyperbolic reflection with respect to the line l: to find the reflection A' of a point A, drop a perpendicular through A to l, call H the foot of that perpendicular. Then A' is the unique point such that H is the midpoint of AA'.

Problem 7. (15 points)

In the Poincare model, you can define circles as in any Hilbert plane (they are sets of points at equal Poincare-distance to the center). Show that such "Poincare-circles" are the same as Euclidean circles.