MAT 319

Practice FINAL

Problem 1. What is the limit of $(x_n) = \frac{n^3}{n!}$?

Problem 2. Use the definition of the limit to prove that $\lim \frac{n^2-1}{3n^2+1} = \frac{1}{3}$.

Problem 3. Prove that an increasing sequence that is bounded above is necessarily converging.

Problem 4. Show that if u_n is unbounded then there is a subsequence u_{n_k} of terms that are all non zero and such that $\frac{1}{u_{n_k}} \to 0$.

Problem 5. Is the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2-n+2}$ convergent?

Problem 6. Evaluate the following limit, or show that it doesn't exist: $\lim_{x\to+\infty} \frac{\sqrt{x}-x^2}{\sqrt{x}+x.\sqrt{x}}$.

Problem 7. Assume that $f\colon \mathbb{R} \to \mathbb{R}$ is such that: for any $x \in \mathbb{R}$ there is a $\delta > 0$ such that f is bounded on $[x-\delta, x+\delta]$. Is the function f bounded on $\mathbb{R}$? (If yes, prove it; if not give a counter-example).

Problem 8. Is the function $g\colon \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 3x + |x|$ differentiable everywhere? (Prove your assertion!)

Problem 9. If $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable at $c \in \mathbb{R}$, then show that
$\lim (n(f(c+\frac{1}{n}) - f(c))$ exists and is equal to $f'(c)$.

Problem 10. Show that if $x > 0$ then we have $\sqrt[3]{1+x} \leqslant 1 + \frac{1}{3}x$