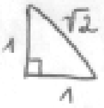
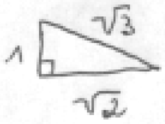
HW 1 Solutions

#1: $AB^2 - AC^2 = (\vec{AB}+\vec{AC})\cdot(\vec{AB}-\vec{AC}) = 2\vec{AM}\cdot\vec{CB}$.

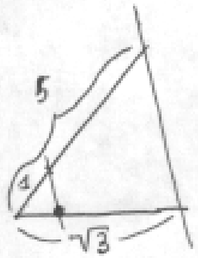
#2:



and then



. Finish with "Thales theorem" (Similitude of triangles):



#3: $\vec{AM}\cdot\vec{AB} = AB^2 \iff (\vec{AB}+\vec{BM})\cdot\vec{AB} = AB^2 \iff AB^2 + \vec{BM}\cdot\vec{AB} = AB^2 \iff \vec{BM}\cdot\vec{AB} = 0$

So $\{M \,/\, \vec{AM}\cdot\vec{AB} = AB^2\} = \{M \,/\, \vec{BM}$ perpendicular to $\vec{AB}\}$ = the line passing through B and perpendicular to AB.

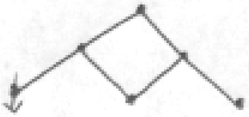
#4: $\vec{MA}\cdot\vec{MB} = 0 \iff (\vec{MI}+\vec{IA})\cdot(\vec{MI}+\vec{IB}) = 0$ (with I midpoint of AB)

$\iff MI^2 + \vec{MI}\cdot\underbrace{(\vec{IA}+\vec{IB})}_{=\vec{0}} - IA^2 = 0$ (because $\vec{IA} = -\vec{IB}$)

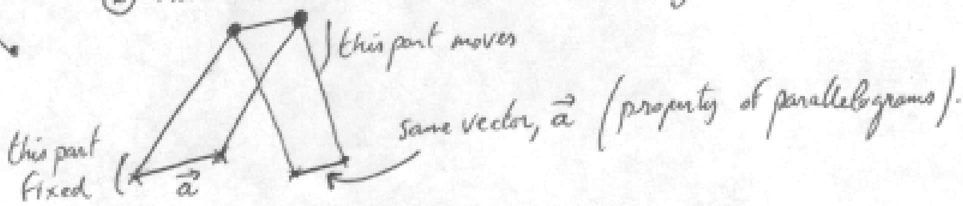
$\iff MI = IA$

So the set τ is the circle with diameter AB.

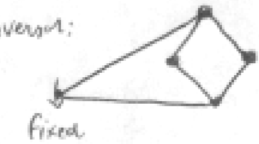
#5: (a) "pantograph"!



(b) How to add a fixed vector $\vec{a}$ to any vector $\vec{OM}$:



(c) inversion:



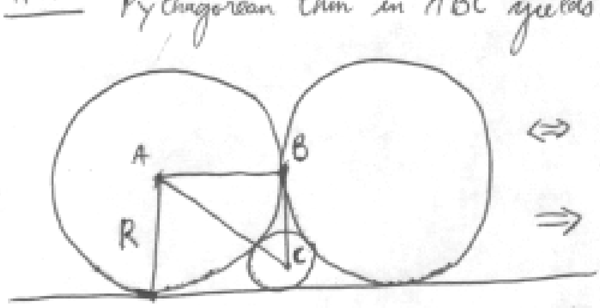
(d) We know how to do $x \mapsto x \pm 1$ and $x \mapsto \frac{1}{x}$,

so compose them: $x \mapsto x-1 \mapsto \frac{1}{x-1}$

Now $\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$ so use (a) to get $\frac{1}{x^2-1}$, apply (c) to get x^2-1

apply (b) to get x^2.

#6: Pythagorean thm in ABC yields:



$$(R+r)^2 = R^2 + (R-r)^2$$

$$\iff \cancel{R^2} + 2rR + \cancel{r^2} = \cancel{R^2} + R^2 - 2rR + \cancel{r^2}$$

$$\Rightarrow 4rR = R^2$$

$$\Rightarrow \boxed{r = \frac{R}{4}}$$