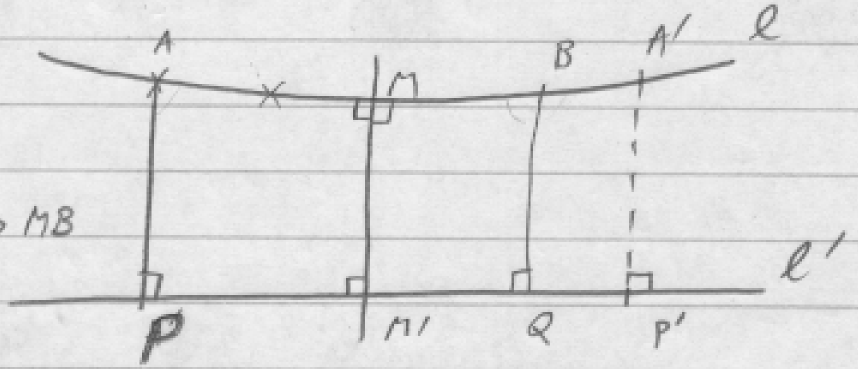
Solutions for HW 7

#1:

There are several cases:



① $A * M * B$:

M is not the midpoint of AB, so assume for example that $AM > MB$

Consider the pt A' on ray $\overrightarrow{MB}$ such that $AM \simeq MA'$, and P' the orthogonal projection of A' onto line l'.

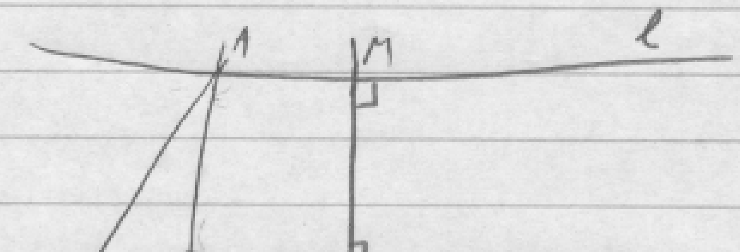
First one must have $AP \simeq A'P'$. (To see this, first show that $AM'A'$ is isosceles, deduce that the angles $\measuredangle AM'M$ and $\measuredangle A'M'M$ are congruent, and then that the angles $\measuredangle AM'P$ and $\measuredangle A'M'P'$ are congruent, and finally using AAS that $\Delta APM' \cong \Delta A'M'P'$).

Therefore we only need to treat the case

② $M * B * A'$

Since the angle $\measuredangle MBQ$ must be acute, the angle $\measuredangle A'BQ$ is obtuse, so is larger than angle $\measuredangle BA'P'$ (which is acute). Therefore, we can use the theorem saying that in any bi-right quadrilateral $BQP'A'$ if $\measuredangle B > \measuredangle A'$ then $A'P' > BQ$.

#2



Let C be the foot of the perpendicular through A to l'.

We will prove $MM' < AC < AB$.

First $AC < AB$ because $\measuredangle B$ is acute and $\measuredangle C$ is right. ("Greater angle correspond to greater opposite side").

Now angle at A must be acute so in the bi-right quadrilateral $AMM'C$ one has $\measuredangle A < \measuredangle M$ thus $MM' < AC$.

#3 See textbook (p251 for 5th edition, and p188 for 3rd edition).