Correction...

Ok, so it seems that there is a divine justice punishing the arrogant professor beginning a sentence like this: "Ok guys, you made many mistakes in your HW, now let me show you how things should be done..."

Now here is what I should have said the other day....

Exercise 1. Given three distinct points such that $A * B * C$, show that ray $\overrightarrow{AB}$= ray $\overrightarrow{AC}$.

Proof. Let's prove that $\overrightarrow{AB} \subset \overrightarrow{AC}$ (the other inclusion is similar).

Consider a point $D \in \overrightarrow{AB}$:

according to the definition of a ray, there are 2 cases to study:

1. ***D* is a point of the segment AB**:

   either a) D is an endpoint of AB ($D = A$ or $D = B$): we are done because these points are already on ray $\overrightarrow{AC}$.

   or b) we have $A * D * B$: then it is enough to show that in this case we have $A * D * C$ or $A * C * D$ (because both mean that D is on the ray $\overrightarrow{AC}$). By axiom B3, we just need to rule out one single case, namely $C * A * D$

   Ruling out $C * A * D$: take a line l intersecting line AB at A (exists because of I3). Now B and C are on the same side of l (otherwise l would intersect the segment BC at A between B and C, impossible because we know that $A * B * C$), and D and B are on the same side of l (same reason, we know that $A * D * B$ is true), thus D and C are on the same side of l and this says that $C * A * D$ is impossible.

2. **D satisfies $A * B * D$**:

   Consider the points A,C, D: by axiom B3 there are only 3 possibilities:

   $A * C * D$ (but in this case we are done, D is on ray $\overrightarrow{AC}$), $A * D * C$ (but in this case we are done, D is on ray $\overrightarrow{AC}$), and $D * A * C$.

   Ruling out $D * A * C$: pick a line m intersecting line AB only at A (exists because I3). Then $A * B * C \Longrightarrow$ B and C are on the same side of m, and $A * B * D \Longrightarrow$ B and D are on the same side. Thus we know that C and D must be on the same side (axiom B4), but this rules out exactly $D * A * C$.

Remark:

when I made a picture of Case 1), it appeared that case CAD was ruled out, but also I tried desperately to rule out ACD (but I didn't need to). Now is it true that ACD must be ruled out, or is it just the picture that is misleading us?

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