Math 311 Homework 3 due on Fr. 02/16

Problem 1. *Characterize the set of positive integers n such that $\phi(2n) > \phi(n)$.*

Problem 2. *What are the last two digits, that is the tens and units digits of* $2^{1000}, 3^{1000}$*?*

Problem 3. *Prove that for $n \geq 2$ the sum of all the positive integers less than n and coprime with n is $\frac{n}{2}.\phi(n)$.*

Problem 4. *Find all the primes p such that p divides $2^p + 1$.*

Problem 5. *Show that $x^2 - 2y^2 + 8z = 3$ has no solutions $(x, y, z) \in \mathbb{Z}^3$. (***Hint:** *reduce modulo 8).*

Problem 6. *For any n show that $\phi(n) = n.(1 - \frac{1}{p_1}) \ldots (1 - \frac{1}{p_k})$ where the p_i are the prime factors present in the prime decomposition of n. (***Hint:***: compute first $\phi(p^s)$, for any prime p.)*